8

REPLICATED DATA

8.1 INTRODUCTION

A replicated database is a distributed database in which multiple copies of some data items are stored at multiple sites. The main reason for using replicated data is to increase DBS availability. By storing critical data at multiple sites, the DBS can operate even though some sites have failed. Another goal is improved performance. Since there are many copies of each data item, a transaction is more likely to find the data it needs close by, as compared to a single copy database. This benefit is mitigated by the need to update all copies of each data item. Thus, Reads may run faster at the expense of slower Writes.

Our goal is to design a DBS that hides all aspects of data replication from users' transactions. That is, transactions issue Reads and Writes on data items, and the DBS is responsible for translating those operations into Reads and Writes on one or more copies of those data items. Before looking at the architecture of a DBS that performs these functions, let's first determine what it means for such a system to behave correctly.

Correctness

We assume that a DBS managing a replicated database should behave like a DBS managing a one-copy (i.e., nonreplicated) database insofar as users can tell. In a one-copy database, users expect the interleaved execution of their
transactions to be equivalent to a serial execution of those transactions. Since replicated data should be transparent to them, they would like the interleaved execution of their transactions on a replicated database to be equivalent to a serial execution of those transactions on a one-copy database. Such executions are called one-copy serializable (or 1SR). This is the goal of concurrency control for replicated data.

This concept of one-copy serializability is essentially the same as the one we used for multiversion data in Chapter 5. In both cases we are giving the user a one-copy view of a database that may have multiple copies (replicated copies or multiple versions) of each data item. The only difference is that here we are abstracting replicated copies, rather than multiple versions, from the users' view.

**The Write-All Approach**

In an ideal world where sites never fail, a DBS can easily manage replicated data. It translates each Read(x) into Read(xA), where xA is any copy of data item x (xA denotes the copy of x at site A). It translates each Write(x) into \{Write(xA), . . . , Write(xA)\}, where \{xA, . . . , xA\} are all copies of x. And it uses any serializable concurrency control algorithm to synchronize access to copies. We call this the write-all approach to replicated data.

To see why the write-all approach works, consider any execution produced by the DBS. Since the DBS is using a serializable concurrency control algorithm, this execution is equivalent to some serial execution. In that serial execution, each transaction that writes into a data item x writes into all copies of x. From the viewpoint of the next transaction in the serial execution, all copies of x were written simultaneously. So, no matter which copy of x the next transaction reads, it reads the same value, namely, the one written by the last transaction that wrote all copies of x. Thus, the execution behaves as though it were operating on a single copy database.

Unfortunately, the world is less than ideal — sites can fail and recover. This is a problem for the write-all approach, because it requires that the DBS process each Write(x) by writing into all copies of x, even if some have failed. Since there will be times when some copies of x are down, the DBS will not always be able to write into all copies of x at the time it receives a Write(x) operation. If the DBS were to adhere to the write-all approach in this situation, it would have to delay processing Write(x) until it could write into all copies of x.

Such a delay is obviously bad for update transactions. If any copy of x fails, then no transaction that writes into x can execute to completion. The more copies of x that exist, the higher the probability that one of them is down. In this case, more replication of data actually makes the system less
available to update transactions! For this reason, the write-all approach is unsatisfactory.

The Write-All-Available Approach

Suppose we adopt a more flexible approach. We still require the DBS to produce a serializable execution, but no longer require a transaction to write into all copies of each data item \( x \) in its writeset. It should write into all of the copies that it can, but it may ignore any copies that are down or not yet created. This is the write-all-available approach. It solves the availability problem, but may lead to problems of correctness.

Using the write-all-available approach, there will be times when some copies of \( x \) do not reflect the most up-to-date value of \( x \). A transaction that reads an out-of-date copy of \( x \) can create an incorrect, i.e., non-1SR, execution. The following execution, \( H_1 \), shows how this can happen:

\[
H_1 = \omega_0[x_A] \omega_0[x_B] \omega_0[y_C] c_0 r_1[y_C] \omega_1[x_A] c_1 r_2[x_B] \omega_2[y_C] c_2.
\]

Notice that \( T_2 \) read copy \( x_B \) of \( x \) from \( T_0 \), even though \( T_1 \) was the last transaction before it that wrote into \( x \). That is, \( T_2 \) read an out-of-date copy of \( x \).

In a serial execution on a one-copy database, if a transaction reads a data item \( x \), then it reads \( x \) from the last transaction before it that wrote into \( x \). But this is not what happened in \( H_1 \). \( T_2 \) read \( x_B \) from \( T_0 \), which is not the last transaction before it that wrote into \( x \). Thus \( H_1 \) is not equivalent to the serial execution \( T_0 \ T_1 \ T_2 \) on a one-copy database. We could still regard \( H_1 \) as correct if it were equivalent to another serial execution on a one-copy database. However, since \( \omega_0[y_C] < r_1[y_C] < \omega_2[y_C] \), there are no other serial executions equivalent to \( H_1 \). Therefore, \( H_1 \) is not equivalent to any serial execution on a one-copy database. That is, \( H_1 \) is not 1SR.

\( T_1 \) seems to be the culprit here, because it did not write into all copies of \( x \). Unfortunately, it may have had no choice. For example, suppose site \( B \) failed after \( T_0 \) but before \( T_1 \), and recovered after \( T_1 \) but before \( T_2 \), as in \( H'_1 \):

\[
H'_1 = \omega_0[x_A] \omega_0[x_B] \omega_0[y_C] c_0 B-fails r_1[y_C] \omega_1[x_A] c_1 B-recovers r_2[x_B] \omega_2[y_C] c_2.
\]

Rather than waiting for \( B \) to recover so it could write \( x_B \), \( T_1 \) wrote into the one copy that it could write, namely, \( x_A \). After \( B \) recovered, \( T_2 \) unwittingly read \( x_B \), an out-of-date copy of \( x \), and therefore produced an incorrect result. This particular problem could be easily solved by preventing transactions from reading copies from sites that have failed and recovered until these copies are brought up-to-date. Unfortunately, this isn't enough, as we'll see later (cf. Section 8.4.).

There are several algorithms, including some variations of the write-all-available approach, that correctly handle failures and recoveries and thereby avoid incorrect executions such as \( H_1 \). These algorithms are the main subject
of this chapter. But before we delve deeply into this subject, let's first define a system architecture for DBSs that manage replicated data.

8.2 SYSTEM ARCHITECTURE

We will assume that the DBS is distributed. As usual, each site has a data manager (DM) and transaction manager (TM) that manage data and transactions at the site.

The Data Manager

The DM is a centralized DBS that processes Reads and Writes on local copies of data items. It has an associated local scheduler for concurrency control, based on one of the standard techniques (2PL, TO, or SGT). In addition, there may be some interaction between local schedulers, for example, to detect distributed deadlocks or SG cycles.

As in Chapter 7, we assume that the DM and scheduler at a site are able to commit a transaction’s Writes that were executed at that site. By committing a transaction $T$, the scheduler guarantees that the recoverability condition holds for all of $T$’s Reads at that site, and the DM guarantees that all of $T$’s Writes at that site are in stable storage (i.e., it satisfies the Redo Rule).

The scheduler at a site is only sensitive to conflicts between operations on the same copy of a data item. For example, if the scheduler at site $A$ receives an operation $r_{[x_A]}$, it will synchronize $r_{[x_A]}$ relative to Writes it has received on $x_A$. However, since it doesn’t receive Writes on other copies of $x$, it cannot synchronize $r_{[x_A]}$ relative to Writes on those other copies. The scheduler is really treating operations on copies as if they were operations on independent data items. In this sense, it is entirely oblivious to data replication.

The Transaction Manager

The TM is the interface between user transactions and the DBS. It translates users’ Reads and Writes on data items into Reads and Writes on copies of those data items. It sends those Reads and Writes on copies to the appropriate sites, where they are processed by the local schedulers and DMs.

The TM also uses an atomic commitment protocol (ACP), so that it can consistently terminate a transaction that accessed data at more than one site. We assume that the DM and scheduler at each site are designed to participate in this ACP for any transaction that is active at that site.

To perform its functions, the TM must determine which sites have copies of which data items. It uses directories for this purpose.¹ There may be just one

¹Note that the term directory has a different meaning here than in Chapter 6. In this chapter, a directory maps each data item $x$ to the sites that have copies of $x$. In Chapter 6, it mapped each data item to its stable storage location.
directory that tells where all copies of all data items are stored. The directory can be stored at all sites or only at some of them. Alternatively, each directory may only give the location of copies of some of the data items. In this case, each directory is normally only stored at those sites that frequently access the information that it contains. To find the remaining directories, the TM needs a master directory that tells where copies of each directory are located.

To process a transaction, a TM must access the directories that tell it where to find the copies of data items that the transaction needs. If communication is expensive, then the directories should be designed so that each TM will usually be able to find a copy of those directories at its own site. Otherwise, the TM will have to send messages to other sites to find the directories it needs.

### Failure Assumptions

We say that a copy $x_A$ of a data item or directory at site $A$ is available to site $B$ if $A$ correctly executes each Read and Write on $x_A$ issued by $B$ and $B$ receives $A$'s acknowledgment of that execution. Thus copy $x_A$ may be unavailable to $B$ for one of three reasons:

1. $A$ does not receive Reads and Writes on $x_A$ issued by $B$. In this case, by definition, a communications failure has occurred (see Section 7.2).
2. The communication network correctly delivers to $A$ Reads and Writes on $x_A$ issued by $B$, but $A$ is unable to execute them, either because $A$ is down or $A$ has suffered a failure of the storage medium that contains $x_A$.
3. $A$ receives and executes each Read and Write issued by $B$, but $B$ does not receive $A$'s acknowledgment of such executions (due to a communications failure).

We say a copy $x_A$ is available (or unavailable) if it is available (or not available) to every site other than $A$.

As in Chapter 7, we assume that sites are fail-stop, and that site failures are detected by timeout. Thus, in the absence of communications failures, each site can determine whether any other site is failed simply by sending a message and waiting for a reply. That is, site failures are detectable.

If communications failures may occur, then a site $A$ that is not responding to messages may still be functioning. This creates nasty problems for managing replicated data, because $A$ may try to read or write its copies without being able to synchronize against Reads or Writes on copies of the same data item at other sites.

### Distributing Writes

When a transaction issues $\text{Write}(x)$, the DBS is responsible for eventually updating a set of copies of $x$ (the exact set depends on the algorithm used for
managing replicated data). It can distribute these Writes immediately, at the moment it receives Write(x) from the transaction. Or, it can defer the Writes on replicated copies until the transaction terminates.

With deferred writing, the DBS uses a nonreplicated view of the database while the transaction is executing. That is, for each data item that the transaction reads or writes, the DBS accesses one and only one copy of that data item. (Different transactions may use different copies.) The DBS delays the distribution of Writes to other copies until the transaction has terminated and is ready to commit. The DBS must therefore maintain an intentions list of deferred updates.2 After the transaction terminates, it sends the appropriate portion of the intentions list to each site that contains replicated copies of the transaction's writeset, but that has not yet received those Writes. It can piggyback this message with the vote-req message of the first phase of the ACP.

Except for the copies it uses while executing the transaction, a DBS that uses deferred writing puts all replicated Writes destined for the same site in a single message. This tends to minimize the number of messages required to execute a transaction. By contrast, using immediate writing, the DBS sends Writes to replicated copies while the transaction executes. Although some piggybacking may be possible, it essentially uses one message for each Write. Thus, immediate writing tends to use more messages than deferred writing.

Another advantage of deferred writing is that Aborts often cost less than with immediate writing. In a DBS that uses immediate writing, when a transaction T, aborts, the DBS is likely to have already distributed many of T,'s Writes to replicated copies. Not only are these Writes wasted, but they also must be undone. With deferred writing, the DBS delays the distribution of those Writes until after T, has terminated. If T, aborts before it terminates, then the abortion is less costly than with immediate writing.

A disadvantage of deferred writing is that it may delay the commitment of a transaction more than immediate writing. This is because the first phase of the ACP at each receiving site must process a potentially large number of Writes before it can respond to the vote-req message. With immediate writing, receiving sites can execute many of a transaction's Writes while the transaction is still executing, thereby avoiding the delay of executing them at commit time.

A second disadvantage of deferred writing is that it tends to delay the detection of conflicts between operations. For example, suppose transactions T, and T2 execute concurrently and both write into x. Furthermore, suppose the DBS uses copy xA while executing T, and uses xB while executing T2. Until the DBS distributes T,'s replicated Write on xB or T2's replicated Write on xA, no scheduler will detect the conflicting Writes between T, and T2. With deferred writing, this happens at the end of T,'s and T2's execution. This may be less desirable than immediate writing, since it may cause a scheduler to

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2Cf. the no-undo/redo centralized recovery algorithm in Section 6.6.
reject a Write later in a transaction’s execution. The DBS ends up aborting the transaction after having paid for most of the transaction’s execution. (This is similar to a disadvantage of 2PL certifiers described in Section 4.4.)

This disadvantage of deferred writing can be mitigated by requiring the DBS to use the same copy of each data item, called the primary copy, to execute every transaction. For example, the DBS would use the same (primary) copy of $x$, $x_A$, to execute both $T_1$ and $T_2$. The scheduler for $x_A$ detects the conflict between $T_1$’s and $T_2$’s writes, thereby detecting it earlier than if $T_1$ and $T_2$ used different copies of $x$. In this case, deferred writing and immediate writing detect the conflict at about the same point in a transaction’s execution.

8.3 SERIALIZABILITY THEORY FOR REPLICATED DATA

We will extend basic serializability theory by using two types of histories: replicated data (RD) histories and one-copy (1C) histories. RD histories represent the DBS’s view of executions of operations on a replicated database. 1C histories represent the interpretation of RD histories in the users’ single copy view of the database. (1C histories are quite similar to the 1V histories we used in Chapter 5.) As usual, we will characterize a concurrency control algorithm by the RD histories it produces. To prove an algorithm correct, we prove that its RD histories are equivalent to serial 1C histories, which are the histories that the user regards as correct.

The formal development of serializability theory for replicated databases is very similar to that for multiversion databases. The notations and the formal notions of correctness are analogous. You may find it helpful to think about these similarities while you’re reading this section.

Replicated Data Histories

Let $T = \{T_0, \ldots, T_n\}$ be a set of transactions. To process operations from $T$, a DBS translates $T$’s operations on data items into operations on the replicated copies of those data items. We formalize this translation by a function $h$ that maps each $r_i[x]$ into $r_i[x_A]$, where $x_A$ is a copy of $x$; each $w_i[x]$ into $w_i[x_{A_1}], \ldots, w_i[x_{A_m}]$ for some copies $x_{A_1}, \ldots, x_{A_m}$ of $x$ ($m > 0$); each $c_i$ into $c_i$ and each $a_i$ into $a_i$.

A complete replicated data (RD) history $H$ over $T = \{T_0, \ldots, T_n\}$ is a partial order with ordering relation $<$ where

1. $H = h(\bigcup_{i=0}^{n} T_i)$ for some translation function $h$;
2. for each $T_i$ and all operations $p_i, q_i$ in $T_i$, if $p_i < q_i$, then every operation in $h(p_i)$ is related by $<$ to every operation in $h(q_i)$;
3. for every $r_i[x_A]$, there is at least one $w_i[x_A] < r_i[x_A]$;
4. all pairs of conflicting operations are related by <, where two operations conflict if they operate on the same copy and at least one of them is a Write; and

5. if \( w_i[x] < r_i[x] \) and \( h(r_i[x]) = r_i[x_A] \) then \( w_i[x_A] \in h(w_i[x]) \).

Condition (1) states that the DBS translates each operation submitted by a transaction into appropriate operations on copies. Condition (2) states that the RD history preserves all orderings stipulated by transactions. Condition (3) states that a transaction may not read a copy unless it has been previously initialized. Condition (4) states that the history records the execution order of noncommutative operations. Condition (5) states that if a transaction \( T_i \) writes into a data item \( x \) before it reads \( x \), then it must write into the same copy \( x_A \) of \( x \) that it subsequently reads. (It may write into other copies of \( x \) as well.) By Condition (2), this implies \( w_i[x_A] < r_i[x_A] \). We will revisit this issue in a moment.

Given transactions \( \{ T_0, T_1, T_2, T_3 \} \):

\[
T_0 = w_i[y] \rightarrow c_0 \quad T_1 = w_i[x] \rightarrow r_i[x] \rightarrow w_2[y] \rightarrow c_1
\]

\[
T_i = r_i[x] \rightarrow w_i[x] \rightarrow c_i \quad T_3 = r_i[y] \rightarrow c_3
\]

the following history, \( H_2 \), is an RD history over \( \{ T_0, \ldots, T_3 \} \):

\[
H_2 = w_i[x_A] \rightarrow r_i[x_B] \rightarrow w_1[x_A] \rightarrow c_1 \rightarrow r_i[x_A] \\
H_2 = w_i[x_B] \rightarrow \quad r_i[x_B] \rightarrow w_i[x_B] \\
H_2 = w_i[y_C] \rightarrow c_i \rightarrow w_i[x_R] \rightarrow r_i[x_R] \rightarrow w_2[y_C] \rightarrow c_1 \\
H_2 = w_i[y_D] \rightarrow r_i[y_D] \rightarrow w_2[y_D] \rightarrow r_i[y_D]
\]

Let \( H \) be an RD history over \( T \). Transaction \( T_j \) reads-x-from \( T_i \) in \( H \) if for some copy \( x_A \), \( T_j \) reads-x,A-from \( T_i \), that is, if \( w_j[x_A] < r_r[x_A] \) and no \( w_k[x_A] \) \((k \neq i)\) falls between these operations. Since reads-from relationships on copies are unique by condition (4) of RD histories and since a transaction reads at most one copy of any given data item,\(^1\) reads-from relationships on data items are unique as well.

In this chapter, we will only consider histories that are complete and that only contain committed transactions (i.e., histories that are committed projections of themselves). These restrictions are justified by requiring that we only

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\(^1\)This follows from the fact that \( h(r_i[x]) = r_i[x_A] \) (i.e., \( h(r_i[x]) \) does not equal a set of Reads) and from the assumption that a transaction reads a data item at most once. As we mentioned in Chapter 2, the latter assumption is mainly a convenience that can be dispensed with at some cost in notational complexity.
use schedulers that produce recoverable executions. Recoverability for RD histories is defined as for 1C histories, but with respect to copies. That is, an RD history \( H \) is recoverable if whenever \( T_i \) reads (any copy) from \( T_j \) in \( H \) and \( c_i \in H \), then \( c_j < c_i \). We assume that all RD histories are recoverable.

The serialization graph for an RD history is defined as for a 1C history. That is, the nodes correspond to committed transactions in the history and there is an edge \( T_i \rightarrow T_j \) if there are conflicting operations \( p_i \) in \( T_i \) and \( q_j \) in \( T_j \) such that \( p_i < q_j \). Thus, the serialization graph for \( H \) is

\[
SG(H) = \begin{array}{c}
\text{T}_0 \\
\downarrow \\
\text{T}_1 \\
\downarrow \\
\text{T}_2
\end{array}
\]

If \( SG(H) \) is acyclic, then conditions (2) and (5) in the definition of RD history ensure that \( H \) preserves reflexive reads-from relationships. More precisely, we have:

\textbf{Lemma 8.1:} Let \( H \) be an RD history involving transaction \( T_i \). If \( SG(H) \) is acyclic and for some \( x \) \( w_i[x] < r_i[x] \), then \( T_i \) reads-\( x \)-from \( T_i \) in \( H \).

\textit{Proof:} From conditions (2) and (5) on RD histories, \( w_i[x] < r_i[x] \) implies that for some copy \( x_A \) of \( x \), \( w_i[x_A] < r_i[x_A] \). Suppose, by way of contradiction, \( T_i \) didn't read \( x \) from \( T_i \) in \( H \). Then there must exist some \( w_k[x_A] \) (\( k \neq i \)) in \( H \) such that \( w_i[x_A] < w_k[x_A] < r_i[x_A] \). But then \( SG(H) \) contains edges \( T_i \rightarrow T_k \) and \( T_k \rightarrow T_i \) which contradicts the assumed acyclicity of \( SG(H) \). \( \Box \)

We would like to define an RD history \( H \) to be 1SR if it is equivalent to a serial 1C history \( H_{1C} \). To determine if \( H \) is equivalent to \( H_{1C} \), it would be unsatisfactory to use the notion of conflict equivalence, simply because \( H \) and \( H_{1C} \) have different operations.\(^4\) However, reads-from relationships and final writes do behave the same way in both types of histories. Therefore, view equivalence provides a natural way to determine the equivalence of an RD history and 1C history (see Section 2.6 for a discussion of view equivalence in 1C histories).

Given RD history \( H \), define \( w_i[x_A] \) to be a \textit{final write for} \( x_A \) in \( H \) if \( a_i \in H \) and for all \( w_j[x_A] \) in \( H \) (\( j \neq i \)), either \( a_j \in H \) or \( w_j[x_A] < w_i[x_A] \).

Two RD histories over \( T \) are \textit{equivalent} (denoted \( \equiv \)) if they are view equivalent, that is, if they have the same reads-from relationships and final writes.

\(^4\)That is, if we define \( H \) and \( H_{1C} \) to be equivalent if conflicting operations appear in the same order in both histories, then we must deal with the problem that some operations that conflict in \( H_{1C} \) may not have corresponding operations that conflict in \( H \). For example, consider \( H = w_2[x_0] w_1[x_A] \) and \( H_{1C} = w_2[x] w_1[x] \).
An RD history $H$ over $T$ is equivalent to a 1C history $H_{1C}$ over $T$ if

1. $H$ and $H_{1C}$ have the same reads-from relationships on data items (i.e., $T_i$ reads-$x$-from $T_j$ in $H$ iff the same holds in $H_{1C}$), and
2. for each final write $w_i[x]$ in $H_{1C}$, $w_i[x_A]$ is a final write in $H$ for some copy $x_A$ of $x$.

An RD history is **one-copy serializable (1SR)** if it is equivalent to a serial 1C history. For example, $H_2$ is 1SR since it is equivalent to the following 1C history:

$H_2 = w_0[x] \ w_0[y] \ c_0 \ w_2[x] \ r_2[x] \ w_2[y] \ c_2 \ r_1[x] \ w_1[x] \ c_1 \ r_1[y] \ c_1.$

History $H_1$ in Section 8.1, reproduced below, is not 1SR:

$H_1 = w_0[x_A] \ w_0[x_B] \ w_0[y_C] \ c_0 \ r_1[y_C] \ w_1[x_A] \ c_1 \ r_1[x_B] \ w_1[y_C] \ c_2.$

Notice that $H_1$ is a serial RD history. Thus, not every serial RD history is 1SR.

**Ignoring Final Writes**

All of the concurrency control algorithms for replicated databases that we will study use conflict-based schedulers. That is, they use schedulers that produce histories that have acyclic serialization graphs. We can use this fact to avoid dealing with final writes in proving the equivalence of RD and 1C histories, as described in the following lemma.

**Lemma 8.2:** Let $H$ be an RD history over $T$, where $SG(H)$ is acyclic. Let $H_{1C}$ be a serial 1C history over $T$ such that the order of transactions in $H_{1C}$ is consistent with $SG(H)$. (That is, if $T_i \rightarrow T_j$ is in $SG(H)$, then $T_i$ precedes $T_j$ in $H_{1C}$.) If $w_i[x]$ is a final write for $x$ in $H_{1C}$, then every Write, $w_i[x_A]$, by $T_i$ into some copy $x_A$ of $x$ is a final write for $x_A$ in $H$.

**Proof:** Suppose $w_i[x]$ is a final write for $x$ in $H_{1C}$. Let $w_i[x_A]$ be any Write into $x$ by $T_i$ in $H$. If $w_i[x_A]$ is not a final write, then there is some $w_j[x_A]$ ($j \neq i$) such that $a_j \not\in H$ and $w_i[x_A] < w_j[x_A]$. Thus, $T_i \rightarrow T_j$ is in $SG(H)$, so $T_j$ precedes $T_i$ in $H_{1C}$. But that implies $a_j \not\in H_{1C}$ and $w_j[x] < w_i[x]$ in $H_{1C}$, contradicting the choice of $w_i[x]$ as a final write.

To prove that an RD history $H$ is equivalent to a serial 1C history $H_{1C}$, we would ordinarily have to prove that $H$ and $H_{1C}$ have the same reads-from relationships and final writes. However, if transactions in $H_{1C}$ are in an order

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1Recall from Section 2.6 that $w_i[x]$ is a **final write for** $x$ in $H_{1C}$ if $a_i \not\in H_{1C}$ and for all $w_j[x] \in H_{1C}$ ($j \neq i$), either $a_j \not\in H$ or $w_j[x] < w_i[x]$. Notice that final writes are defined for Writes on data items in 1C histories and for Writes on copies of data items in RD histories. Hence the need for the special wording of condition (2) in the definition of equivalence of $H$ and $H_{1C}$, instead of simply saying they have the same reads-from relationships and final writes.
consistent with $SG(H)$ (which, since $H_{\text{C}}$ is serial, implies that $SG(H)$ is acyclic), then by Lemma 8.2 we only have to prove they have the same reads-from relationships. That is, we have the following theorem.

**Theorem 8.3:** Let $H$ be an RD history. If $H$ has the same reads-from relationships as a serial 1C history $H_{\text{C}}$, where the order of transactions in $H_{\text{C}}$ is consistent with $SG(H)$, then $H$ is 1SR.

### 8.4 A GRAPH CHARACTERIZATION OF 1SR HISTORIES

To prove the correctness of a concurrency control algorithm for replicated data, we must prove that all of the histories it produces are 1SR. Following our usual approach, we would like to do this with the help of a serialization graph structure. Unfortunately, standard serialization graphs for RD histories are too weak for this purpose, as illustrated by the following example.

Consider a database with data items $x$ and $y$ and copies $x_A$, $x_B$, $y_C$, and $y_D$. Suppose we have the following three transactions:

- $T_0 = r_0[x] \rightarrow w_0[x] \rightarrow c_0$
- $T_1 = r_1[x] \rightarrow w_1[y] \rightarrow c_1$
- $T_2 = r_2[y] \rightarrow w_2[x] \rightarrow c_2$

If the DBS uses 2PL and ignores failed copies, the following history can occur.

- $H_4 = w_0[x_A] \rightarrow r_1[x] \rightarrow y_D \rightarrow w_1[y_C] \rightarrow c_1$
- $H_4 = w_0[x_B] \rightarrow r_2[y] \rightarrow x_A \rightarrow w_2[x_B] \rightarrow c_2$

where $x_A \rightarrow$ denotes the failure of copy $x_A$. We'll find it more convenient to speak of copies, rather than sites, failing. Transactions $T_1$ and $T_2$ begin by reading $x_A$ and $y_D$. After they complete their Reads, the copies that they read fail. Then they perform their Writes. Since $y_D$ is down, $y_C$ is the only available copy of $y$. So the DBS translates $w_1[y]$ into $w_1[y_C]$. Similarly, since $x_A$ is down, it translates $w_2[x]$ into $w_2[x_B]$. If the DBS uses 2PL, by setting locks on copies that it accesses, it has no trouble locking each copy that $T_1$ and $T_2$ access, because no two operations of these transactions access the same copy of any data item.

$H_4$ is not 1SR. A serial execution of $T_0$, $T_1$, and $T_2$ on a single copy database would have either $T_1$ reading the value of $x$ written by $T_2$, or $T_2$ reading the value of $y$ written by $T_1$. However, in $H_4$ neither transaction reads the data written by the other. So, $H_4$ is not equivalent to a serial history over $\{T_0, T_1, T_2\}$ on a one-copy database. But despite the fact that $H_4$ is incorrect, $SG(H_4)$ is acyclic:

$$SG(H_4) = T_0 \rightarrow T_1 \rightarrow T_2.$$
This example illustrates the rather surprising fact that the write-all-available approach may lead to an incorrect execution, even if only failures, but no recoveries occur (i.e., failed copies never recover).

One explanation of the problem in $H$, relates to the fundamental technique that all concurrency control algorithms use to obtain correct executions, namely, controlling the order of conflicting operations on shared data. In $H$, even though $T_i$ and $T_j$ have conflicting accesses to data items $x$ and $y$, they don't have conflicting accesses to copies of $x$ and $y$. The copy of $x$ and $y$ that each of them read failed before the other transaction's conflicting Write into that copy could be issued. The copy that each of them wrote is a copy that the other transaction didn't read. Thus, the logical conflicts were never manifested as physical conflicts on copies, which is the only place that the DBS can control them.

In a sense, the problem is finding a way to ensure that any two transactions that have conflicting accesses to the same data item also have conflicting accesses to some copy of that data item. That way, the DBS will be able to synchronize the transactions. If the DBS can do this in all cases, then by attaining ordinary serializability it has also attained one-copy serializability.

*Replicated Data Serialization Graphs*

To determine if an RD history is 1SR, we will use a modified SG. This graph models the observation that two transactions that have conflicting accesses to the same data item must be synchronized, even if they don't access the same copy of that data item. To define this graph, we need a little terminology. We say that node $n_i$ precedes node $n_j$, denoted $n_i < n_j$, in a directed graph if there is a path from $n_i$ to $n_j$.

Given an RD history $H$, a replicated data serialization graph (RDSG) for $H$ is $SG(H)$ with enough edges added (possibly none) such that the following two conditions hold: For all data items $x$,

1. if $T_i$ and $T_k$ write $x$, then either $T_i < T_k$ or $T_k < T_i$,

2. if $T_j$ reads $x$ from $T_n$, $T_k$ writes some copy of $x$ ($k \neq i$, $k \neq j$), and $T_j \neq T_k$, then $T_j < T_k$.

If a graph satisfies condition (1), we say it induces a write order for $H$. If it satisfies condition (2), we say it induces a read order for $H$. Thus RDSG($H$) is an extension of $SG(H)$ that induces a read order and a write order for $H$. Note that an RDSG for $H$ isn't uniquely determined by $H$.

We explained that failures can lead to incorrect behavior because transactions that have conflicting accesses to the same data item may not have conflicting accesses to copies of that data item. An SG doesn't have enough edges (due to conflicting accesses to copies) to force an order on every pair of transactions with conflicting accesses to the same data item. An RDSG for $H$ adds enough edges to $SG(H)$ to make this so. The write order ensures that the
RDSG orders every pair of transactions that write into the same data item, even if they don't write into the same copy. If the RDSG induces a write order, then the read order ensures that the RDSG orders every pair of transactions \( (T_j \text{ and } T_k) \) that (respectively) read and write the same data item.

\( \text{SG}(H_4) \) is not an RDSG\( (H_4) \). It induces a write order for \( H_4 \), but not a read order. We can make it into an RDSG\( (II_4) \) by adding edges between \( T_1 \) and \( T_1 \).

\[
G = T_0 \xrightarrow{} T_1 \xrightarrow{} T_2
\]

Since \( T_1 \) reads-x-from \( T_0 \), \( T_2 \) writes x, and \( T_0 \rightarrow T_2 \), we added \( T_1 \rightarrow T_2 \). Since \( T_2 \) reads-y-from \( T_0 \), \( T_1 \) writes y, and \( T_0 \rightarrow T_1 \), we added \( T_1 \rightarrow T_1 \). Since these are the only two reads-from relationships in \( H_4 \), these two edges are enough to ensure that \( G \) induces a read order for \( H \). Notice that \( G \) has a cycle. Since every RDSG for \( H_4 \) must contain the edges \( T_1 \rightarrow T_2 \) and \( T_1 \rightarrow T_1 \), every such RDSG has a cycle.

The following theorem is an important tool for analyzing the correctness of concurrency control algorithms for replicated data.

**Theorem 8.4:** Let \( H \) be an RD history. If \( H \) has an acyclic RDSG, then \( H \) is 1SR.

**Proof:** Let \( H_s = T_{i_1}, T_{i_2}, \ldots, T_{i_t} \) be a serial 1C history where \( T_{i_1}, T_{i_2}, \ldots, T_{i_t} \) is a topological sort of RDSG\( (H) \). Since RDSG\( (H) \) contains SG\( (H) \), by Theorem 8.3, we can prove that \( H \) is 1SR just by proving that \( H \) and \( H_s \) have the same reads-from relationships.

First, assume \( T_j \) reads-x-from \( T_i \) in \( H \). Suppose, by way of contradiction, that \( T_j \) reads-x-from \( T_k \) in \( H \), for some \( k \neq i \). If \( k = j \), Lemma 8.1 implies that \( T_j \) reads-x-from \( T_k \) in \( H \), a contradiction (note that since \( H \) has an acyclic RDSG, SG\( (H) \) is surely acyclic and therefore Lemma 8.1 applies). So, assume \( k \neq j \). Since \( T_j \) reads-x-from \( T_i \) in \( H \), \( T_i \rightarrow T_j \) is in the RDSG of \( H \), so \( T_i \) precedes \( T_j \) in \( H \). Since the RDSG induces both a read and a write order, we have that either \( T_k \ll T_i \) or \( T_j \ll T_k \). Thus either \( T_k \) precedes \( T_i \) (which precedes \( T_j \)) or \( T_k \) follows \( T_j \) in \( H \), both contradicting that \( T_j \) reads-x-from \( T_k \) in \( H \).

Now assume \( T_j \) reads-x-from \( T_i \) in \( H_i \). By conditions (3) and (4) in the definition of RD history and the definition of reads-from, \( T_j \) reads-x-from some transaction in \( H \), say \( T_h \). By the previous paragraph, \( T_j \) reads-x-from \( T_h \) in \( H \). Since read-from relationships are unique, \( T_h = T_j \).

**8.5 ATOMICITY OF FAILURES AND RECOVERIES**

Another characterization of 1SR histories is that they are RD histories in which failure and recovery events appear to be atomic. That is, in a 1SR history, all transactions have a consistent view of when copies fail and recover. In the next
two subsections, we will explain this characterization by means of examples. Then we will describe a graph structure that captures this characterization.

Atomicity of Failures

Loosely speaking, a transaction learns about a failure when it tries to read or write a copy that turns out to be unavailable. Similarly, it knows that a copy could not have failed yet if it successfully accesses that copy. Depending on when each transaction learns about failures, different transactions might see failures occurring in different orders.

For example, reconsider history $H_z$.

\[ H_z = \begin{align*}
&\rightarrow r_1[x_A] \rightarrow y_D \rightarrow w_1[y_c] \rightarrow c_1 \\
&\rightarrow r_1[y_D] \rightarrow y_A \rightarrow w_2[x_B] \rightarrow c_2 \\
&\rightarrow w_0[x_A] \\
&\rightarrow r_0[x_B] \\
&\rightarrow w_0[y_B] \\
&\rightarrow w_1[y_c] \\
&\rightarrow w_1[y_D]
\end{align*} \]

Since $T_1$ decided not to write $y_D$, it must have learned about $y_D$'s failure before it committed; this means that in an equivalent serial execution, $y_D$ failed before $T_1$ executed. And since it read $x_A$, it believes $x_A$ failed after it executed. Thus, $T_1$ sees failures in the following order: $y_D$ + $T_1$ + $x_A$. By contrast, $T_2$ sees the failures in the opposite order. It didn't write $x_A$, so it must have learned that $x_A$ failed before it ran. But it read $y_D$, so it believes $y_D$ failed after it ran. Thus, it sees $x_A$ + $T_2$ + $y_D$.

Suppose we think of failures as atomic events that must be recorded in a serial execution. Given $T_1$'s and $T_2$'s view of these events, there is no serial execution of $T_1$ and $T_2$ in which we can place the failure events $x_A$ and $y_D$. For example, if the given execution were equivalent to the serial execution $T_1 T_2$, then $T_1 \rightarrow x_A \rightarrow T_2$ is consistent with $T_1$'s and $T_2$'s view. But $T_2$ thinks $y_D$ failed before $T_1$, and $T_2$ thinks $y_D$ failed after $T_2$. Both views cannot be true if $y_D$ is an atomic event. Instead, if the given execution were equivalent to $T_2 T_1$, then we have $T_2 \rightarrow y_D \rightarrow T_1$, but now we have inconsistent views of $x_A$ to contend with.

To ensure that failure events appear to be atomic, we need to synchronize failures with Reads and Writes. Unfortunately, failures are not controllable events. They happen whenever they want to. So, the best we can do is ensure that transactions see failures in a consistent order. This, as it turns out, is all we need.

Note that a transaction can see a failure by not issuing a Write on an unavailable copy. Therefore, the DBS must not only control the order in which transactions read and write copies, but must also control the transaction's decision whether or not to issue a Write on a copy. Thus, the TM’s translation activity has an effect on one-copy serializability.
Atomicity of Recoveries

To create a new copy of \( x \), or to recover a formerly failed copy of \( x \), say \( x_B \), the DBS must store an initial value in \( x_B \). Any transaction that ordinarily writes into \( x \) can initialize \( x_B \). Whenever it writes into other copies of \( x \), it writes into \( x_B \) too. Rather than waiting for such a transaction to appear, the DBS can force the initialization of \( x_B \) by running a special transaction, called a *copier*. The copier simply reads an existing copy of the data item that is up-to-date and writes that value into the new copy. Whether the DBS uses an ordinary transaction or a copier to initialize \( x_B \), it must ensure that no transaction reads \( x_B \) until it has been initialized.

The DBS must also make \( x_B \) known to all transactions that update \( x \), so that they will write into \( x_B \) whenever they write into other copies of \( x \). This latter activity requires some synchronization, as the following example illustrates.

Suppose the database has copies \( x_A \) and \( y_C \) at the time that the DBS is ready to create copy \( x_B \). It runs a copier transaction, \( T_1 \), to initialize \( x_B \) at about the same time that two other transactions \( T_2 \) and \( T_3 \) execute. The resulting execution is as follows.

\[
H_s = \begin{array}{c}
\text{\( w_0[x_A] \)} \\
\text{\( c_0 \)} \\
\text{\( w_0[y_C] \)} \\
\end{array}
\quad
\begin{array}{c}
\text{\( r_1[x_A] \rightarrow w_1[x_B] \rightarrow c_1 \rightarrow r_2[x_B] \rightarrow c_3 \)} \\
\text{\( r_2[x_A] \rightarrow w_2[x_A] \rightarrow c_3 \rightarrow r_3[y_C] \)} \\
\text{\( w_3[y_C] \)} \\
\end{array}
\]

\( H_s \) is incorrect. The only serial RD history equivalent to \( H_s \) is

\[
H'_s = \begin{array}{c}
\text{\( w_0[x_A] \)} \\
\text{\( w_0[y_C] \)} \\
\end{array}
\quad
\begin{array}{c}
\text{\( c_0 \)} \\
\text{\( c_0 \rightarrow r_1[x_A] \rightarrow w_1[x_B] \rightarrow c_1 \rightarrow r_2[x_B] \rightarrow c_3 \)} \\
\text{\( r_2[x_A] \rightarrow w_2[x_A] \rightarrow c_3 \rightarrow r_3[y_C] \)} \\
\text{\( w_3[y_C] \)} \\
\end{array}
\]

\( H'_s \) is not equivalent to the serial 1C history \( T_0 T_1 T_2 T_3 \). In a one-copy database, \( T_2 \) would write \( x \) and \( y \), and \( T_3 \) would read the values that \( T_2 \) wrote. But in this execution, \( T_3 \) read the value of \( x \) that \( T_1 \) wrote and the value of \( y \) that \( T_3 \) wrote. \( H_s \) is not equivalent to any other serial 1C history either, and so is not 1SR.

The problem is that \( T_1 \) should have updated the new copy of \( x \), \( x_B \), but didn't. Since \( T_1 \) knows that \( x_B \) exists (it wrote \( x_B \)), and since \( T_1 \) effectively executes before \( T_2 \) (because \( r_1[x_A] < w_2[x_A] \)), \( T_2 \) should also know that \( x_B \) exists. Therefore, since \( T_2 \) writes into \( x \), it should write all copies of \( x \), including \( x_B \). If it did, then \( r_3[x_B] \) would have read the proper value.

We can also explain this in terms of the atomicity of recovery events. Let us denote the recovery (or initialization) of \( x_A \) as \( x_A \sqcup \). Since \( T_1 \) wrote \( x_B \), it believes \( x_B \) recovered before it executed. That is, \( x_B \sqcup \rightarrow T_1 \). Since \( T_2 \) wrote \( x_A \) but not \( x_B \), it believes \( x_B \) recovered after it executed. That is, \( T_2 \rightarrow x_B \sqcup \). But since \( T_1 \rightarrow T_2 \) is in \( \text{SG}(H_s) \), these views of the recovery of \( x_B \) are inconsistent.\(^6\)

\(^6\)An alternative analysis is \( x_B \sqcup \rightarrow T_1 \rightarrow x_B \sqcap \rightarrow T_2 \rightarrow x_B \sqcup \rightarrow T_3 \). We have \( x_B \sqcup \rightarrow T_1 \) because \( T_2 \) read \( x_B \). But now \( T_3 \) reads the recovered copy \( x_B \) before it has been initialized (with the value of \( x \) written by \( T_2 \)), which is illegal.
*Failure-Recovery Serialization Graphs*

As we have seen, another explanation why a serializable execution may not be 1SR is that different transactions observe failures and recoveries in different orders. We can formalize this reasoning by augmenting SGs to include nodes that represent the creation and failure of each copy. To keep the notation simple, we will assume that each copy is created once and sometime later fails. *Once it has failed, a copy never recovers.* This assumption is not a loss of generality because we can regard the recovery of a copy as the creation of a new copy. That is, a copy that fails and recovers several times is represented by a sequence of uniquely named incarnations, each of which is created, fails at most once, and never recovers. An alternative model for incarnations is suggested in Exercise 8.9.

Given an RD history $H$ over transactions $\{T_0, \ldots, T_n\}$, a failure-recovery serialization graph (FRSG) for $H$ is a directed graph with nodes $N$ and edges $E$ where:

$$\begin{align*}
N &= \{T_0, \ldots, T_n\} \cup \{\text{create}[x_A] \mid x \text{ is a data item and } x_A \text{ is a copy of } x\} \\
&\quad \cup \{\text{fail}[x_A] \mid x \text{ is a data item and } x_A \text{ is a copy of } x\} \\
E &= \{T_i \rightarrow T_j \mid T_i \text{ reads or writes } x_A\} \cup E1 \cup E2 \cup E3
\end{align*}$$

where

$$\begin{align*}
E1 &= \{\text{create}[x_A] \rightarrow T_i \mid T_i \text{ reads or writes } x_A\} \\
E2 &= \{T_i \rightarrow \text{fail}[x_A] \mid T_i \text{ reads } x_A\} \\
E3 &= \{T_i \rightarrow \text{create}[x_A] \text{ or fail}[x_A] \rightarrow T_i \mid T_i \text{ writes some copy of } x, \text{ but not } x_A\}
\end{align*}$$

The edges in $E1$ signify that if $T_i$ read or wrote $x_A$, then $x_A$ was created before $T_i$ executed ($\text{create}[x_A] \rightarrow T_i$). $E2$ signifies that if $T_i$ read $x_A$, then $T_i$ executed before $x_A$ failed ($T_i \rightarrow \text{fail}[x_A]$). Notice that this need not hold if $T_i$ wrote (but did not read) $x_A$. $E3$ signifies that if $T_i$ wrote some copies of $x$ but did not write $x_A$, then it must have written those copies at a time when $x_A$ did not exist, that is, either before it was created or after it failed. Thus, for each such situation, $E3$ must contain either $T_i \rightarrow \text{create}[x_A]$ or $\text{fail}[x_A] \rightarrow T_i$. As in RDSGs, $H$ does not uniquely determine an FRSG.

For example, the following is an FRSG for $H_5$.

```
create[x_A] --- T_1 ----> fail[x_A]  
create[x_B] ---- T_0 ----> fail[x_B]  
create[x_C] --> T_2 ----> fail[x_C] 
create[x_D] 
```

Since $T_1$ wrote $x_C$, but not $x_D$, we must include either $T_1 \rightarrow \text{create}[x_D]$ or $\text{fail}[x_D] \rightarrow T_1$; we chose the latter. Similarly, we added $\text{fail}[x_A] \rightarrow T_2$, because $T_1$ wrote $x_B$ but not $x_A$. Notice that this FRSG has a cycle: $T_1 \rightarrow \text{fail}[x_A] \rightarrow T_2 \rightarrow \text{fail}[y_D] \rightarrow T_1$. In fact, *every* FRSG for $H_5$ has a cycle.
8.6 AN AVAILABLE COPIES ALGORITHM

Given a history \( H \), if there is an acyclic \( \text{FRSG}(H) \), then we can produce a serial history that is equivalent to \( H \) and which includes the creation and failure of copies as atomic events. That is, all transactions observe the creation and failure of each copy in the same order. This condition is enough to show that \( H \) is 1SR.

**Theorem 8.5:** Let \( H \) be an RD history. If \( H \) has an acyclic \( \text{FRSG} \), then \( H \) is 1SR.

**Proof:** As in Theorem 8.4, let \( H_s = T_i, T_{i-1}, \ldots, T_1 \) be a serial 1C history where \( T_i, T_{i-1}, \ldots, T_1 \) is a topological sort of \( \text{FRSG}(H) \). Since \( \text{FRSG}(H) \) contains \( \text{SG}(H) \), by Theorem 8.3 we can prove that \( H \) is 1SR just by proving that \( H \) and \( H_s \) have the same reads-from relationships.

First assume \( T_j \) reads-xA-from \( T_i \) in \( H \). Hence \( T_i \rightarrow T_j \) is in the FRSG and \( T_i \) precedes \( T_j \) in \( H \). Let \( T_k \) be any other transaction that writes \( x \). If \( T_k \) writes \( x_A \), then since \( T_j \) reads-xA-from \( T_i \), either \( T_k \rightarrow T_i \) or \( T_i \rightarrow T_k \) must be in the FRSG. If \( T_k \) does not write \( x_A \), by definition of \( \text{FRSG} \), either \( T_k \rightarrow \text{create}[x_A] \) or \( T_k \rightarrow \text{fail}[x_A] \rightarrow T_k \). In the former case, since \( \text{create}[x_A] \rightarrow T_i, T_k \) precedes \( T_i \) in the FRSG. In the latter case, since \( T_j \rightarrow \text{fail}[x_A], T_j \) precedes \( T_k \) in the FRSG. Hence, if \( T_k \) writes \( x \), either \( T_k \) precedes \( T_i \) or follows \( T_j \) in the FRSG and in \( H_s \), too. Thus, \( T_j \) reads-x-from \( T_i \) in \( H_s \).

Now, suppose \( T_j \) reads-x-from \( T_i \) in \( H_s \). By the definition of RD history, \( T_j \) reads-x-from some transaction in \( H \), say \( T_h \). By the previous paragraph, \( T_j \) reads-x-from \( T_h \) in \( H_s \). Since reads-from relationships are unique, \( T_h = T_i \).

**8.6 AN AVAILABLE COPIES ALGORITHM**

Available copies algorithms handle replicated data by using enhanced forms of the write-all-available approach. That is, every \( \text{Read}(x) \) is translated into a Read of any copy of \( x \) and every \( \text{Write}(x) \) is translated into Writes of all available copies of \( x \). Simply doing this is not enough to guarantee one-copy serializability, as history \( H_s \) showed. Available copy algorithms enforce special protocols that, in conjunction with this "read-any, write-all-available" discipline, ensure correctness.

These algorithms handle site failures but not communications failures. That is, they assume that every site is either operational or down, and that all operational sites can communicate with each other. Therefore, each operational site can independently determine which sites are down, simply by attempting to communicate with them. If a site doesn't respond to a message within the timeout period, then it must be down.

For available copies algorithms, we'll assume that the scheduler uses strict two phase locking. Thus, after transaction \( T_i \) has read or written a copy of \( x_A \), no other transaction can access \( x_A \) in a conflicting mode until after \( T_i \) has committed or aborted.
In this section we'll describe a simple available copies algorithm. We assume that there is a fixed set of copies for each data item, known to every site. This set does not change dynamically. Moreover, to keep the description simple, we'll initially assume that each copy is created and fails at most once. Later we'll discuss how to accommodate repeated failures and recoveries of copies — no changes to the algorithm are needed for this extension! After a copy has been initialized and before it has failed, it is said to be available; otherwise, it is unavailable.

Processing Reads and Writes

When a transaction $T_i$ issues a Read($x$), the TM at $T_i$'s home site (henceforth simply “$T_i$’s TM”) will select some copy $x_A$ of $x$ and submit Read($x_A$) on behalf of $T_i$ to site $A$. Typically the selected copy will be the one “closest” to $T_i$'s home site (ideally the home site itself), to minimize the communication cost incurred by the Read. The correctness of the algorithm, however, does not depend on which copy is read — any one will do.

Site $A$ regards $x_A$ as initialized if it has already processed a Write on $x_A$, even if the transaction that issued the Write has not yet committed. If site $A$ is operational and $x_A$ is initialized, Read($x_A$) will be processed by the scheduler and DM at $A$. If $x_A$ is initialized but the transaction $T_i$ that initialized $x_A$ has not yet committed, then by Strict 2PL the scheduler must delay Read($x_A$) until $T_i$ commits or aborts. If Read($x_A$) is rejected, a negative acknowledgment will be returned and $T_i$ will be aborted. If Read($x_A$) is accepted, the value read will be returned to $T_i$’s TM and Read($x$) will be done. However, if site $A$ is down or if $x_A$ hasn’t been initialized, then $T_i$’s TM will eventually time out waiting for a response. In that event the TM could abort $T_i$ or, better, it could submit Read($x_B$) to another site $B$ that contains a copy of $x$. As long as one of the copies of $x$ can be read, Read($x$) will be successful. If no copy of $x$ can be read, $T_i$ must abort.

When $T_i$ issues a Write($x$) operation, its TM sends Write($x_A$) operations to every site $A$ where a copy of $x$ is supposed to be stored. If $A$ is down, Write($x_A$) will not be received and, of course, its processing will never be acknowledged. $T_i$’s TM will eventually time out waiting for a response from $A$. If $A$ is operational, the handling of Write($x_A$) depends on whether or not $x_A$ was previously initialized.

If $x_A$ has been initialized, then Write($x_A$) must be processed by $A$’s scheduler and DM, and eventually a response is returned to $T_i$’s TM indicating whether Write($x_A$) was rejected or duly processed.

---

7Recall from Section 7.1 that $T_i$’s “home site” is the site where it originated and whose TM supervises its execution.

8It is pointless to submit a Read($x_B$) for a different copy $x_B$ of $x$, as the conflict that caused the rejection of Read($x_A$) will also cause the rejection of Read($x_B$).
On the other hand, if \( x_A \) has not been initialized, the DBS at \( A \) has two options. It could use \( \text{Write}(x_A) \) to initialize \( x_A \) at this time (in which case the operation is processed as just described), or it may ignore \( \text{Write}(x_A) \), preferring to not initialize \( x_A \) yet. Ignoring the operation means that \( A \) doesn't send an acknowledgment to \( T_i \)'s TM, that is, \( A \) acts as if it were down as far as \( x_A \) is concerned.\(^9\)

After sending \( \text{Writes} \) to all of \( x \)'s copies, \( T_i \)'s TM waits for responses. It may receive rejections from some sites, positive responses from others (meaning the Write has been accepted and performed), and no responses from others (those that have failed or that have not initialized their copies of \( x \)). Writes for which no responses are received are called missing writes. If any rejection is received or if all \( \text{Writes} \) to \( x \)'s copies are missing, then \( \text{Write}(x) \) is rejected and \( T_i \) must abort. Otherwise, \( \text{Write}(x) \) is successful.

**Validation**

So far we’ve described, in some detail, the “read-one, write-all-available” discipline. We know from history \( H_i \) that this isn’t enough to guarantee one-copy serializability. The available copies algorithm uses a validation protocol to ensure correctness. Transaction \( T_i \)'s validation protocol starts after \( T_i \)'s Reads and \( \text{Writes} \) on copies have been acknowledged or timed out. At that time \( T_i \) knows all its missing writes as well as all the copies it has actually accessed (read or written). The validation protocol consists of two steps:

1. **missing writes validation**, during which \( T_i \) makes sure that all copies it tried to, but couldn’t, write are still unavailable, and

2. **access validation**, during which \( T_i \) makes sure that all copies it read or wrote are still available.

It is important that missing writes validation be performed before access validation (see Exercise 8.11).

To validate missing writes, \( T_i \) sends a message \( \text{UNAVAILABLE}(x_A) \) to site \( A \), for each copy \( x_A \) that \( T_i \) found unavailable. \( A \) will acknowledge such a message only if it has, in the meanwhile, initialized \( x_A \) (i.e., received and processed a \( \text{Write}(x_A) \), even if the Write has not yet been committed).

After sending the \( \text{UNAVAILABLE} \) messages, \( T_i \) waits for responses. At the end of the timeout period, if it has not received any acknowledgments to these messages, it proceeds with access validation. Otherwise, some copy it hasn’t updated has been initialized and \( T_i \) is aborted. At this point it is important to recall the assumption that there are no communication failures. It implies that if \( T_i \) has not received any acknowledgment to the \( \text{UNAVAILABLE} \) messages by

\(^9\)Alternatively, \( A \) might send a message expressly indicating that \( x_A \) has not been initialized yet. This will prevent \( T_i \)'s TM from waiting for the full timeout period before concluding that \( x_A \) is not available.
the end of the timeout period, then it must be that no such acknowledgment was sent and, therefore, that all copies \( T_i \) couldn’t write are still unavailable.\(^\text{10}\)

If the missing writes validation step succeeds, then \( T_i \) proceeds with access validation. To that end, \( T_i \) sends a message \( \text{AVAILABLE}(x_A) \) to site \( A \), for each copy \( x_A \) that \( T_i \) read or wrote. \( A \) acknowledges this message if \( x_A \) is still available at the time \( A \) receives the message. If all \( \text{AVAILABLE} \) messages are acknowledged, then access validation succeeds and \( T_i \) is allowed to commit. Otherwise \( T_i \) must abort.

The validation protocol requires a significant amount of communication and is therefore expensive. We can reduce the number of messages sent by combining all \( \text{UNAVAILABLE} \) messages from \( T_i \) to some particular site into one message. Similarly for \( \text{AVAILABLE} \) messages and acknowledgments. Even so, we will have two steps (missing writes validation and access validation), each requiring two rounds of message exchanges (one for the \( \text{UNAVAILABLE} / \text{AVAILABLE} \) messages and one for acknowledgments). Fortunately, things aren’t as bad as this may suggest.

First, if a transaction has no missing writes, there is no need for missing writes validation! Thus, after all copies have been initialized and in the absence of (site) failures, the first step of the validation protocol is avoided.

Second, access validation can be combined with atomic commitment. Recall that when \( T_i \) terminates, its TM sends \( \text{VOTE-REQ} \) messages to all sites where \( T_i \) accessed copies. The \( \text{VOTE-REQ} \) message sent to a site can be used as an implicit \( \text{AVAILABLE} \) message. If a site responds \( \text{YES} \), surely all copies accessed by \( T_i \) at that site are still available; thus a \( \text{YES} \) response can be used as an implicit acknowledgment for \( \text{AVAILABLE} \). Therefore, access validation can ride for free on the coattails of atomic commitment.\(^\text{11}\)

**Some Examples**

To develop some insight on how the validation protocol ensures one-copy serializability, we consider some examples of non-1SR executions and show how they are prevented from happening. In the next subsection we prove the correctness of the algorithm.

Let’s start with history \( H_i \). We reproduce it next, embellished with certain new symbols to represent events of interest. For transaction \( T_i \), \( t_i \) represents the

\(^{10}\)It is possible that a copy was initialized after \( T_i \) attempted to write it and failed before \( T_i \) started its missing writes validation step. The proof of the algorithm’s correctness, which will be presented shortly, should quell any concern that such behavior could compromise one-copy serializability.

\(^{11}\)Under certain circumstances the atomic commitment protocol needn’t involve sites where a transaction only read but did not write (cf. Section 7.3). However, access validation requires that messages be sent to such sites, to verify that copies read are still available. Thus, some additional cost to atomic commitment may be incurred by access validation.
moment when \( T_i \) begins its access validation step. By the specification of the algorithm, \( t_i \) must follow all Read and Write operations of \( T_i \) as well as the missing writes validation step (if present), and must precede \( c_i \). As usual, the symbol \( x_A \square \) stands for the failure of \( x_A \).

\[
H_i' = \begin{array}{c}
w_0[x_A] \\
w_0[x_B] \\
w_0[y_B] \\
r_1[x_A] \rightarrow r_1[y_B] \\
t_1 \rightarrow c_1 \\
r_2[y_C] \rightarrow w_2[x_B] \\
t_2 \rightarrow c_2 \\
x_A \square \\
y_C \square \\
\end{array}
\]

Consider the precedences involving the failure and access validation events. Since \( T_i \) read \( x_A \), the failure of \( x_A \) must have occurred after \( T_i \) started its access validation. Otherwise \( T_i \) would have found \( x_A \) to be unavailable and would therefore abort. Therefore, \( t_i < x_A \square \) and, similarly, \( t_2 < y_C \square \).

Since \( T_i \) wrote \( y_D \) but not \( y_C \), it must have carried out missing writes validation and found that \( y_C \) is still unavailable. Since at that time \( y_C \) had already been initialized, it must be that \( y_C \) failed before the completion of \( T_i \)'s missing writes validation and thus before the beginning of access validation. Hence, \( y_C \square < t_1 \). The precedence \( x_A \square < t_2 \) is justified on similar grounds.

Given these precedences we have a cycle in \( H_i' \): \( t_i < x_A \square < t_2 < y_C \square < t_1 \). This is impossible since \( H_i' \), being a history, is supposed to be a partial order. This means that \( H_i' \) could not have happened.

In terms of the algorithm, the reason why \( H_i' \) couldn't have happened is this: If \( T_i \)'s access validation succeeded we'd have \( t_i < x_A \square < t_2 \), so \( T_i \)'s access validation started after \( T_i \)'s. Since \( y_C \square < t_2 \), we get \( y_C \square < t_2 \). But then \( T_2 \)'s access validation would not have succeeded, so \( T_2 \) would have aborted. Similarly, if \( T_i \)'s validation succeeded, \( T_i \)'s would fail. (Of course, both transactions' validations could have failed.)

Note the pivotal role of validation in the justification of why \( H_i' \) couldn't occur. Without missing writes and access validation, we wouldn't be able to assert the existence of the precedences that yield the cycle in \( H_i' \).

Our next example illustrates the significance of the assumption that the scheduler uses Strict 2PL. History \( H_6 \) is an execution of \( T_1, T_2 \) (the same transactions as in \( H_4 \)) where there is only one copy of \( y, y_C \). For the moment ignore the broken arrow from \( c_2 \) to \( w_1[y_C] \).

\[
H_6 = \begin{array}{c}
w_0[x_A] \\
w_0[x_B] \\
w_0[y_C] \\
r_1[x_A] \rightarrow r_1[y_C] \rightarrow t_1 \rightarrow c_1 \\
r_2[y_C] \rightarrow w_2[x_B] \\
t_2 \rightarrow c_2 \\
x_A \square \\
y_C \square \\
\end{array}
\]

Like \( H_6 \), \( H_6 \) is not 1SR; \( T_1 \) doesn't read the value of \( x \) written by \( T_2 \), and \( T_2 \) doesn't read the value of \( y \) written by \( T_1 \), one of which would have to hold in any serial one-copy execution over \( T_1, T_2 \). Unfortunately, the validation steps don't prevent either \( T_1 \) or \( T_2 \) from committing, as you can easily verify. What stops \( H_6 \) from happening is the scheduler's use of Strict 2PL. Since \( T_2 \) read \( y_C \)
before $T_i$ wrote into it, $T_j$ must have locked $y_C$ before $T_i$ did. But then $T_j$ won’t lock $y_C$ until after $T_i$ has committed; i.e., we have $c_2 < \omega_i[y_c]$ (a stronger precedence than just $r_i[y_c] < \omega_i[y_c]$). Now we get a cycle in $H_s$, meaning that $H_s$ couldn’t have occurred. Note that the precedence $r_i[y_c] < \omega_i[y_c]$ isn’t enough to give rise to a cycle. It’s important that $T_j$ keeps its locks until it commits.

*Proof of Correctness*

We’ll state seven properties that are satisfied by histories representing executions of the available copies algorithm. We’ll justify why each property is satisfied and then show that any history satisfying them is ISR, thereby proving that the available copies algorithm is correct.

In addition to the symbols we used in the previous examples, we’ll use the symbol $t_i$ to denote the point at which $T_i$ begins its missing writes validation step. Hence all Read and Write operations of $T_i$ must precede $t_i$ and $t_i < t_i$. We’ll also use the symbol $x_A \sqcup$ to denote the creation (initialization) of copy $x_A$. $x_A$ is created when the first transaction that writes it, say $T_i$, begins its access validation. Thus $x_A \sqcup$ is a “synonym” for $t_i$.

The properties that must be satisfied by any history $H$ produced by the available copies algorithm are as follows

- **$AC_i$:** For every $T_i$, $t_i < t_i < c_i$. And for every Read or Write $p_i$ of $T_i$, $p_i < t_i$.
- **$AC_j$:** $SG(H)$ is acyclic.
- **$AC_k$:** If $T_i \rightarrow T_j$ is in $SG(H)$ then $t_i < t_j$.

$AC_i$ defines the execution and validation phases of a transaction. $AC_j$ is satisfied because the scheduler that controls access to copies produces serializable executions. $AC_i$ is satisfied because the scheduler uses Strict 2PL. If $T_i \rightarrow T_j$ is an edge in $SG(H)$, then there exist conflicting operations $p_i[x_A] < q_j[x_A]$ in $H$. By Strict 2PL, $c_i < q_j[x_A]$. Consequently, $t_i < t_j$ (because, by $AC_i$, $t_i < c_i$ and $q_j[x_A] < t_j$).\textsuperscript{12}

- **$AC_r$:** For any $r_i[x_A]$ in $H$, $x_A \sqcup < r_i[x_A]$.
- **$AC_s$:** For any $\omega_i[x_A]$ in $H$, either $x_A \sqcup = t_i$ or $x_A \sqcup < \omega_i[x_A]$.

$AC_r$ and $AC_s$ say that a copy must be initialized before it may be read or written. This is obviously satisfied by the available copies algorithm (cf. the subsection on Processing Reads and Writes).

- **$AC_w$:** If $r_i[x_A]$ or $\omega_i[x_A]$ is in $H$ then $t_i < x_A \sqcup$.

\textsuperscript{12}At the beginning of this section we said that available copies algorithms require a Strict 2PL scheduler. That’s not quite true. The correct statement is that these algorithms require a scheduler that will guarantee $AC_i$ through $AC_s$.\textsuperscript{12}
ACₜ says that Tᵢ's access validation begins before the failure of any copy read or written by Tᵢ. This is satisfied because otherwise one of Tᵢ's AVAILABLE messages would not be acknowledged and so Tᵢ would be aborted.

ACᵣ: If Tᵢ writes into some copy of x but not xₐ then either xₐ∩ < tᵢ or, for any wᵢ[xₐ] in H, tᵢ < wᵢ[xₐ].

To understand ACᵣ, consider the missing writes validation step, in which Tᵢ sent UNAVAILABLE(xₐ) to site A. If A had responded to the message, then Tᵢ would have aborted. So assume not. Thus, either xₐ failed before A received the message (in which case xₐ∩ < tᵢ) or xₐ had not yet been initialized (in which case tᵢ < wᵢ[xₐ] for all Tⱼ that write xₐ).

ACᵣ is where we use the fact that missing writes validation completes before access validation begins. Without this handshake, the two validation steps would execute concurrently, so tᵢ and tᵢ' would not be distinguishable events. If xₐ failed before A received UNAVAILABLE(xₐ), we could conclude that xₐ∩ < cᵢ, but not that xₐ∩ < tᵢ' = tᵢ, which is needed in the correctness proof.

We begin the proof with a preliminary lemma that strengthens ACᵣ.

Lemma 8.6: Let H be a history produced by the available copies algorithm. In H, if Tᵢ writes some copy of x but not xₐ, then either xₐ∩ < tᵢ or tᵢ < xₐ⊔.

Proof: Suppose Tᵢ writes into x₉ but not xₐ. By ACᵣ, either xₐ∩ < tᵢ or tᵢ < w₀[xₐ], where T₀ is the transaction that initialized xₐ. In the former case, H obviously satisfies the lemma. Therefore let us assume the latter and consider two cases, depending on whether or not T₀ wrote into x₉.

CASE 1: Suppose w₀[x₉] is in H. We claim that w₁[x₉] < w₀[x₉] in H. For, otherwise, we'd have w₀[x₉] < w₁[x₉] and, by AC₁, t₀ < t₁. By assumption, tᵢ < w₀[xₐ]. By AC₁, w₀[xₐ] < t₀, so by transitivity tᵢ < t₀, a contradiction. Therefore, w₁[x₉] < w₀[x₉]. By AC₁, t₁ < tᵢ. Since t₀ < t₀ by AC₁, and t₀ = xₐ⊔ by assumption, we have tᵢ < xₐ⊔, as desired.

CASE 2: Suppose T₀ doesn't write x₉. By ACᵣ, either (a) x₉∩ < t₀ or (b) t₀ < w₁[x₉]. If (a), we have tᵢ < x₉∩ by ACₙ, so tᵢ < t₀ = x₉⊔, and we get tᵢ < x₉⊔, as desired. If (b), we have tᵢ < w₀[xₐ] by assumption, w₀[xₐ] < t₀ by AC₁, t₀ < w₁[x₉] (this is (b)), and w₁[x₉] < tᵢ by AC₁. All these imply tᵢ < tᵢ', a contradiction.

Any history that satisfies AC₁ - ACᵣ is an RD history. Conditions (1), (2), and (4) in the definition of RD history are immediate. Condition (3), which requires each rᵢ[xₐ] to be preceded by a wᵢ[xₐ], follows ACₙ. Condition (5) says that if wᵢ[x] <ᵢ rᵢ[x] and rᵢ[xₐ] ∈ H, then wᵢ[xₐ] ∈ H. To see this, suppose wᵢ[xₐ] ∈ H. Then by Lemma 8.6, either (a) xₐ∩ < tᵢ or (b) tᵢ < xₐ⊔. ACₙ contradicts
(a). By AC, \( x_A \uparrow < r_i[x_A] \), and by AC, \( r_i[x_A] < t_i \); thus \( x_A \downarrow < t_i \), which contradicts (b). Hence, \( w_i[x_A] \in H \) as desired.

**Theorem 8.7:** The available copies algorithm produces only 1SR histories.

**Proof:** Let \( H \) be a history that satisfies AC - AC. We claim \( H \) is 1SR. Since \( H \) is an RD history, by Theorem 8.5 it's enough to show that \( H \) has an acyclic FRSG. From \( H \) construct a graph \( G \) with nodes \( N \) and edges \( E \), where:

\[
N = \{ T_i | T_i \text{ appears in } H \} \cup \{ \text{create}[x], \text{fail}[x] | x_A \text{ is a copy of some data item} \} \\
E = \{ T_i \rightarrow T_j | T_i \rightarrow T_j \text{ is an edge of } \text{SG}(H) \} \cup F1 \cup F2 \cup F3 \cup F4 \\
F1 = \{ \text{create}[x] \rightarrow T_i | x_A \downarrow < t_i \text{ or } x_A \downarrow = t_i \} \\
F2 = \{ T_i \rightarrow \text{fail}[x] | t_i < x_A \uparrow \} \\
F3 = \{ \text{fail}[x] \rightarrow T_i | x_A \downarrow < t_i \} \\
F4 = \{ T_i \rightarrow \text{create}[x] | t_i < x_A \downarrow \}
\]

To show that \( G \) is an FRSG, we must show that it contains the edges E1-E3 in the definition of FRSG.

\[
E1 = \{ \text{create}[x] \rightarrow T_i | T_i \text{ reads or writes } x_A \} \\
E2 = \{ T_i \rightarrow \text{fail}[x] | T_i \text{ reads } x_A \} \\
E3 = \{ T_i \rightarrow \text{create}[x] \text{ or fail}[x] \rightarrow T_i | T_i \text{ writes some copy of } x \text{ but not } x_A \}
\]

By AC, if \( T_i \) reads \( x_A \), then \( t_i < x_A \uparrow \), so \( E2 \subseteq F2 \). By Lemma 8.6, if \( T_i \) writes \( x \) but not \( x_A \), then either \( x_A \uparrow < t_i \) or \( t_i < x_A \downarrow \), so some \( E3 \) is contained in \( F3 \cup F4 \).

To show \( G \) contains E1, suppose \( T_i \) reads \( x_A \). By AC, \( x_A \downarrow < r_i[x_A] \), and by AC, \( r_i[x_A] < t_i \). Hence, \( x_A \downarrow < t_i \). If \( T_i \) writes \( x_A \), then by AC, either \( x_A \downarrow = t_i \) or \( x_A \downarrow < \omega_i[x_A] \). In the latter case, by AC, \( \omega_i[x_A] < t_i \), so \( x_A \downarrow < t_i \). Thus \( E1 \subseteq F1 \), and \( G \) contains an FRSG for \( H \).

We now show that \( G \) is acyclic. Define a mapping \( f \) by \( f(\text{create}[x_A]) = x_A \downarrow \), \( f(\text{fail}[x]) = x_A \uparrow \), and \( f(T_i) = t_i \). If \( n_j \rightarrow n_k \) is in \( G \), then either \( f(n_j) < f(n_k) \) in \( H \) or \( f(n_j) = f(n_k) = x_A \downarrow \) for some \( x_A \) (i.e., \( n_j = \text{create}[x_A] \) and \( n_k = t_i = \text{create}[x_A] \); see F1). Thus, for any cycle \( P = n_i, \ldots, n_m, n_i \) in \( G \) \((m > 1)\), there is a corresponding sequence \( P' = f(n_i) \leq \ldots \leq f(n_m) \leq f(n_i) \) in \( H \). Clearly, \( P \) must contain at least one edge \( n_j \rightarrow n_k \) not in \( F1 \). For each such edge, \( f(n_j) < f(n_k) \), by definition of F2 - F4. Therefore, \( f(n_i) < f(n_i) \) in \( P \), contradicting that \( H \) is a partial order. Thus, \( G \) is acyclic.

Since \( G \) contains an FRSG for \( H \), that FRSG is acyclic too. Thus, by Theorem 8.5, \( H \) is 1SR.
Repeated Failures and Recoveries

So far we have assumed that each copy is initialized and fails at most once. This assumption results in a notational simplification — that for each copy $x_A$, we have at most one $x_A \cup$ and one $x_A \cap$ symbol. The usual justification for this assumption is that one can view a single copy that’s initialized and fails repeatedly as a sequence of copies, each of which is created and fails once.

This view might appear to contradict another basic assumption of the available copies algorithm, namely, that the set of copies is fixed. Fortunately, as long as all copies of a data item don’t fail, the “contradiction” is a red herring (see Exercise 8.13). Though the set of copies is fixed, it does not have to be finite!

In particular, for each site $A$ where there is a copy of $x$ we can imagine we have an infinite supply of such copies, $x_A^0, x_A^1, x_A^2, \ldots$. At any time at most one of these copies is available. If copy $x_A^i$ is the one presently available, then copies $x_A^{i-1}, \ldots, x_A^0$ have failed and copies $x_A^{i+1}, x_A^{i+2}, \ldots$ are uninitialized. When (and if) $x_A^i$ fails, $x_A^{i+1}$ will be the next copy of $x$ at site $A$ to be initialized.

Since there is a finite number of sites and at most one copy of $x$ is available at any one of them, Write($x$) results in the writing of a finite number of copies — a relief! Read($x$) involves just one copy, so there is no problem here. Access validation involves the copies read or written — a finite number, as we just argued.

Missing writes validation, however, must ensure that an infinite number of copies is unavailable. Since these copies are stored at a finite number of sites, we can do this with a finite number of messages. The only problem is that now any transaction that writes some data item $x$ will have missing writes (in fact an infinite number of them!), even if it writes copies at all sites where $x$ is stored. Does this mean that every transaction must perform missing writes validation? If this were so, it would negate our earlier assertion that in the absence of failures the available copies algorithm incurs no validation cost at all. Fortunately, it isn’t so. The reason is that access validation can now be used implicitly for missing writes validation. That is, if $T_j$ writes copy $x_A^i$ and during access validation it finds $x_A^i$ is still available, it implicitly knows that copies $x_A^i, x_A^{i+1}, x_A^{i+2}, \ldots$ are still unavailable. Thus there is no need for separate missing writes validation in this case. Of course, missing writes validation is required, as before, if $T_j$ couldn’t write any copy of $x$ at site $A$.

8.7 DIRECTORY-ORIENTED AVAILABLE COPIES

The static assignment of copies to sites in the available copies algorithm of the previous section is a serious disadvantage. It requires, among other things, that transactions attempt to update copies at down sites. If site failures persist for
long periods, this is clearly inefficient. In addition, it is not possible to dynamically create or destroy copies at new sites.

In this section we study the directory-oriented available copies algorithm, which rectifies these problems. The algorithm uses directories to define the set of sites that currently stores the copies of an item. More precisely, for each data item $x$ there is a directory $d(x)$ listing the set of $x$'s copies. Like a data item, a directory may be replicated, that is, it may be implemented as a set of directory copies, stored at different sites.

The directory for $x$ at site $U$, denoted $d_U(x)$, contains a list of the copies of $x$ and a list of the directory copies for $x$ that site $U$ believes are available. For notational clarity, we will typically use \{U, V\} for names of sites that store directories, and \{A, B, C, D\} for those that store data item copies. These sets of sites need not be disjoint. Usually, a site will store both directory and data item copies. We assume that before we begin executing transactions, all copies of directories exist, but that no copies of data items exist. That is, we assume that new directory copies are never created. A method for creating directory copies appears in a later subsection.

Directories are treated like ordinary data items by the DBS. In particular, concurrent access to directory copies is controlled by the same scheduler that controls concurrent access to data item copies. The only difference is that ordinary transactions can only read directories. Directories are updated by two special transactions, Include (or IN) for creating new data item copies and Exclude (or EX) for destroying data item copies.

**Basic Algorithm**

When a site $A$ containing $x$ recovers from failure, or when $A$ wants to create a new copy of $x$, the DBS runs a transaction IN($x_A$). IN($x_A$) brings the value of $x_A$ up-to-date by:

1. finding a directory copy $d_U(x)$, for example, by using a local copy that it knows exists, by reading a master directory that lists copies of $d(x)$, or by polling other sites;
2. reading $d_U(x)$ to find an available copy of $x$, say $x_B$;
3. reading $x_B$ and
4. copying $x_B$'s value into $x_A$.

Thus, it performs the function of a copier. If $d_U(x)$ says that there are no copies of $x$ and never were any, then $A$ should provide an initial value for $x_A$, the first

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[13] As for the simple available copies algorithm, we are assuming a Strict 2PL scheduler or, to be somewhat more general, that transactions that access a data item copy in conflicting modes start their validation protocols in the order in which they accessed that copy.
copy of $x$. In any case, it declares $x_A$ to be available by adding $x_A$ to each available copy of $d(x)$.

When a site fails, some DBS that tries to access data at that site observes the failure. Based on directories it has read, the DBS believes that certain copies are stored at the failed site. Therefore, for each such copy, it runs an EX transaction for each copy stored there. EX($x_A$) declares $x_A$ to be unavailable by removing $x_A$ from every available copy of $d(x)$. That is, it

1. reads some directory copy $d_U(x)$,
2. removes $x_A$ from the list of available copies it read from $d_U(x)$, and
3. writes that updated list into each directory copy listed in $d_U(x)$.

Notice that if a DBS incorrectly believes a copy $x_A$ was at the failed site when in fact it wasn't, it does no harm by executing EX($x_A$).

Let NX be any IN or EX transaction. NX($x_A$) begins by reading a directory, say $d_U(x)$. It expects to be able to update all of $d(x)$’s copies listed in $d_U(x)$. However, some of these directory copies may have recently failed and are therefore unavailable. Not only is NX unable to update such directory copies, but it now knows that the list of available directories in the (remaining) available copies of $d(x)$ directories is wrong. NX corrects such errors as follows.

After reading $d_U(x)$, NX attempts to access the directory copies listed in $d_U(x)$. Let AD be the directory copies listed in $d_U(x)$ that it determines are available (including $d_U(x)$). NX then updates each directory copy in AD by modifying its list of available data item copies and updating its list of available directory copies to be AD. This distribution of directory updates can be overlapped with the first phase of its ACP, by sending the directory update and VOTE-REQ in the same message.

To process Read(x) on behalf of a user transaction, the DBS reads a copy of $d(x)$, say $d_U(x)$. If it tries to read a directory copy that is unavailable, then it simply ignores the attempt and keeps trying other copies until it finds one that is available. It then selects a copy $x_A$ of $x$ that $d_U(x)$ says is available and issues Read($x_A$). Since a failure can happen at any time, it is possible that $d_U(x)$ says that $x_A$ is available, but the DBS discovers that $x_A$ is unavailable when it tries to read it. In this case, the DBS can take one of three actions: (1) it can select another copy that $d_U(x)$ says is available and try to read that copy; (2) it can read a different copy of $d(x)$, $d_V(x)$, and try to read a copy that $d_V(x)$ says is available; or (3) it can abort the transaction.

To process Write(x), the DBS reads a copy of $d(x)$, $d_U(x)$, and issues Write($x_A$) for every copy $x_A$ that $d_U(x)$ says is available. If the DBS discovers

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14If $d_U(x)$ says that there are no copies of $x$ but that there were some in the past, then $x$ is recovering from a total failure. In this case, $A$ can initialize $x$ using its last committed value for $x$ only if $x_A$ was among the last copies of $x$ to have failed. It can determine this fact using the technique of Section 7.5. For pedagogical clarity, we will not incorporate this complexity in the remainder of this section. See Exercise 8.13).
that any copy that \( d_U(x) \) says is available is actually unavailable, it must abort the transaction \( T_i \) that issued the Write and run an EX(\( x_A \)) transaction. When that commits, it can try running \( T_i \) again.\(^{15}\)

After performing all its operations, transaction \( T_i \) must carry out its validation protocol. Since there are never missing writes, missing writes validation is unnecessary. The reason there are no missing writes is that the set of available copies for \( x \) is listed in \( d(x) \) and, as we just saw, unless all these copies are actually written, \( T_i \) will abort. Thus only access validation is needed in the directory-oriented available copies algorithm (another advantage over the simpler algorithm of the preceding section). As we have seen, access validation can be done together with atomic commitment.

In fact, in the directory-oriented algorithm, access validation can be done merely by checking that the directory copies that \( T_i \) read still contain the data item copies \( T_i \) accessed (see Exercise 8.15). Thus, if directory copies are stored at all sites, access validation requires no communication at all.\(^{16}\)

This is especially important if the DBS uses deferred Writes. The DBS cannot do Read validation until the transaction terminates, which doesn’t occur until all Writes are distributed. If the DBS at site \( A \) can validate \( T_i \) only using directory copies stored at \( A \), then it can do access validation locally. This avoids another round of communication after the distribution of \( T_i \)’s Writes.

**Correctness Argument**

We can argue that this algorithm produces 1SR executions by using Theorem 8.5. The theorem says that if all transactions observe creations and failures in the same order, then the execution is 1SR. We can see that every execution of the algorithm satisfies this property by the following intuitive line of reasoning. We will show in a moment that user transactions behave as if the creation or failure of a copy occurred at the moment its IN or EX executed. All transactions, including INS and EXes, effectively execute in the serialization order. Thus, all user transactions see the same order of INS and EXes, and hence see the same order of creations and failures. This is the condition of Theorem 8.5, so the execution is 1SR.

A transaction \( T_i \) demonstrates its belief that a copy \( x_A \) is available by operating on \( x_A \). If it operated on \( x_A \) then it must have read some directory copy \( d_U(x) \) that said that \( x_A \) was available. IN(\( x_A \)) must have written \( x_A \) into \( d_U(x) \)

---

\(^{15}\)A fancier (and better) way to do this is to have \( T_i \) incorporate EX(\( x_A \))’s actions before proceeding. More precisely, \( T_i \) will execute EX(\( x_A \)) as a subtransaction and will commit only if EX(\( x_A \)) commits. Since this gets us into nested transactions, a topic not discussed in this book, we’ll stick with the brute force method of aborting \( T_i \), running EX(\( x_A \)), and then restarting \( T_i \) from scratch.

\(^{16}\)Of course, the atomic commitment protocol must still be carried out. However, now it need not involve sites at which the transaction only read copies.
8.7 DIRECTORY-ORIENTED AVAILABLE COPIES

A transaction $T_i$ demonstrates itsbelief that a copy $x_A$ is not available by not operating on that copy when it should, namely when it writes into data item $x$. When $T_i$ writes $x$, it decides which copies to write by reading one of $x$'s directories, say $d_U(x)$, and writing into all of the copies listed there. If $x_A$ wasn't in $d_U(x)$ at the time the transaction read it, then $T_i$ does not write $x_A$, thereby observing the failure of $x_A$. One way this could occur is if $T_i$ read $d_U(x)$ before IN$(x_A)$ wrote $x_A$ into $d_U(x)$. The other possibility is that EX$(x_A)$ wrote $d_U(x)$ to delete $x_A$ before $T_i$ read $d_U(x)$. In the former case, $T_i$ executed before IN$(x_A)$ and in the latter case itexecuted after EX$(x_A)$. This too is consistent with the view that the copy was created when IN executed and failed when EX executed. Thus, transactions behave as if each IN (or EX) executes at the moment the copy is created (or fails), so, by Theorem 8.5, the execution is 1SR. This argument can be formalized along the lines of the proof given in the previous section for the simpler AC algorithm (see Exercise 8.16).

Creating Directories

To create a directory copy $d_U(x)$, we must store the current list of copies of $x$ and directory copies for $x$ in $d_U(x)$. We must also add $d_U(x)$ to the list of directories in every other directory for $x$. Thus, a Directory Include (or DIN) transaction for $d_U(x)$ begins by reading an existing directory copy for $x$, $d_V(x)$. Like an IN or EX, it checks to see which directory copies, AD, in $d_V(x)$ are still available. Then it updates the directory copies. It sets the list of data item copies in $d_U(x)$ to be that of $d_V(x)$. And it writes AD into the list of directory copies in each directory copy in AD (including $d_U(x)$).

How does a DIN know where to look for an existing copy of $d(x)$? If there is a copy of $d(x)$ at the site that is executing the DIN, then it can easily find the copy. If not, then it could poll other sites, to find out which of them have a copy.

If it can't find a copy, then it should try to initialize the first copy of $d(x)$. Unfortunately, it can't assume that it is the only DIN that is trying to initialize $d(x)$. The reason is that some other DIN may have been polling sites at the same time and reached the same conclusion that $d(x)$ was uninitialized. If both DINs try to create the initial copy of $d(x)$, neither copy will include a reference to the other. This could ultimately lead to an inconsistent state of $x$. For example, each subsequent IN would include copies of $x$ that were only listed in one directory copy or the other. Then a user transaction would update only some copies of $x$, namely, those listed in the directory copy it read. It is therefore essential that only one DIN initialize a first copy of $d(x)$.
Another bad outcome will occur if \( d(x) \) doesn't currently exist because it has experienced a total failure. That is, there used to be copies of \( d(x) \), but they have all failed. It would be incorrect to initialize a new copy of \( d(x) \), because there may still be copies of \( x \) around that are listed in the last copies of \( d(x) \) to have failed.

One way to ensure that a DIN can correctly initialize the first copy of a directory is to have a master directory that contains a list of all directories for which an initial directory has been created and initialized. A DIN that initializes a directory \( d(x) \) must update the master directory. If two DINs try to initialize \( d(x) \) concurrently, they will have conflicting accesses to the master directory, so one DIN will have to wait until the other terminates. The second DIN will therefore see that it is not really the initial DIN for \( d(x) \), and will act accordingly. Moreover, a total failure of \( d(x) \) will be detectable, because the master directory will say that \( d(x) \) exists, but no available site will have a copy.

Clearly, if the master directory is unavailable, then no directories can be initialized. The master directory should therefore be replicated, and all copies updated by each initial DIN. To avoid unbounded recursion, some additional restrictions are needed, such as fixing the set of master directory copies for all time. We leave the algorithms for including and excluding copies of the master directory as an exercise (see Exercise 8.18).

### 8.8 Communication Failures

The main problem with available copies algorithms is that they do not tolerate communication failures. That is, if two or more operational sites cannot communicate, they may produce executions that are not 1SR.

For example, suppose there are copies \( \{x_A, x_B, x_C\} \) of \( x \) and \( \{y_B, y_C, y_D\} \) of \( y \), but a communication failure has partitioned the network into two independent components, \( P_1 = \{A, B\} \) and \( P_2 = \{C, D\} \). That is, component \( P_1 \) has copies \( \{x_A, x_B, y_B\} \) and \( P_2 \) has copies \( \{x_C, y_C, y_D\} \). Suppose transactions \( T_1 \) and \( T_2 \) execute in components \( P_1 \) and \( P_2 \) (respectively) using an available copies algorithm, producing history \( H_\cdot \).

\[
H_\cdot =
\begin{align*}
&\overset{w_0[x_A]}{r_1[y_B]} - w_1[x_A] - w_1[x_B] - c_i \\
&\overset{w_0[x_B]}{r_1[y_B]} - w_1[x_A] - w_1[x_B] - c_i \\
&\overset{w_0[x_C]}{r_1[y_B]} - w_1[x_A] - w_1[x_B] - c_i \\
&\overset{w_0[y_B]}{r_1[y_B]} - w_1[x_A] - w_1[x_B] - c_i \\
&\overset{w_0[y_C]}{r_1[y_B]} - w_1[x_A] - w_1[x_B] - c_i \\
&\overset{w_0[y_D]}{r_1[y_B]} - w_1[x_A] - w_1[x_B] - c_i \\
\end{align*}
\]

In \( P_1 \), \( T_1 \) reads the one and only available copy of \( y \), namely \( y_B \), and writes both available copies of \( x \), \( x_A \) and \( x_B \). Similarly for \( T_2 \) in \( P_2 \). Since each transaction can validate that the copies it accessed were still available at the time it
terminated, each can commit. Yet the execution is clearly not 1SR, since neither transaction reads the output of the other.

The general problem is that transactions in different components may have conflicting accesses on different copies of the same data item, as in $H_7$ just shown. The serialization order of such transactions is essential to the one-copy serializability of their execution (cf. Section 8.4 and Theorem 8.4). But since DBSs executing in different components cannot communicate, they cannot synchronize transactions that execute in different components, and thus cannot ensure one-copy serializability. Therefore, nearly all techniques for handling communications failures focus on preventing transactions that access the same data item from executing in different components.\footnote{See Exercise 8.19 for an approach that allows certain conflicting transactions to execute in different components.}

**Site Quorums**

Surely one way to prevent conflicting transactions from executing in different components is to insist that only one component be allowed to process any transactions at all. Since the components can't communicate, each component must be able to independently decide whether it is the component that can process transactions. One way to accomplish this is to use site quorums.

In site quorums, we assign a non-negative weight to each site. Every site knows the weight of all sites in the network. A quorum is any set of sites with more than half the total weight of all sites (cf. Section 7.5). Only the one component that has a quorum of sites can process transactions.

Unfortunately, it is possible that no component has a quorum. This can occur if the network splits into more than two components, or if sites fail in the component that would have contained a quorum. If this happens, then no component can process transactions. The DBS fails totally.

One should assign weights to sites based on their relative importance to the enterprise that is using the DBS. A site with higher weight is more likely to be in the quorum component.

We can improve on the basic site quorum rule by allowing certain transactions to execute in components that don't have a quorum of sites. Since a nonreplicated data item can only be accessed in the component that has its one and only copy, there cannot be two transactions that access a nonreplicated data item but execute in different components. Therefore, a transaction can safely read and write nonreplicated data in any component. For the same reason, a transaction can read and write any replicated data item $x$ in a component that has all of the replicated copies of $x$. Thus, the site quorum rule need only apply to transactions that access a replicated data item $x$ in a component that is missing one or more copies of $x$.\footnote{See Exercise 8.19 for an approach that allows certain conflicting transactions to execute in different components.}
The major problem with site quorums is that if transactions have inconsistent views of the components, then they can produce incorrect results. For example, given a database containing \( \{x_A, x_B, x_C\} \), suppose site A's TM executes \( T_1 \) believing that only A and B are accessible, while site C's TM executes \( T_2 \) believing that only B and C are accessible, thereby producing \( H_s \).

\[
H_s = \begin{align*}
&\omega_0[x_A] \rightarrow r_1[x_A] \rightarrow w_1[x_A] \rightarrow w_1[x_B] \rightarrow c_1 \\
&\omega_0[x_B] \rightarrow c_0 \\
&\omega_0[x_C] \rightarrow r_2[x_C] \rightarrow w_2[x_B] \rightarrow w_2[x_C] \rightarrow c_2
\end{align*}
\]

TM_A and TM_C both believed they had access to a majority of the sites. But due to a communication anomaly, A and C were each able to communicate with B without being able to communicate with each other. Alternatively, \( H_s \) might have occurred due to the following sequence of events.

1. A disconnects from B and C.
2. TM_C executes \( T_2 \).
3. B disconnects from C and connects to A.
4. TM_A executes \( T_1 \).

We'll look at two solutions to this problem: the quorum consensus algorithm, which requires each transaction to access a quorum of copies of data items on which it operates (Section 8.9), and the virtual partition algorithm, in which each site synchronizes its view of its component with other sites in its component (Section 8.10). The quorum consensus algorithm has less overhead to handle failures and recoveries than the virtual partition algorithm, but has more overhead to process transactions during periods in which no failures or recoveries take place. Thus, the former is more suitable for environments where failures and recoveries are frequent, and the latter for those where they are relatively rare.

**Queries**

Since queries (i.e., read-only transactions) do not modify the database, it would seem to be safe to execute them in all components. After all, since each component uses a correct concurrency control algorithm, each query will read a database state produced by a serializable execution of the transactions in the same component. Curiously, however, the execution across two partitions may not be 1SR.

For example, consider the following history:

\[
H_s = \begin{align*}
&\omega_0[x_A] \\
&\omega_0[y_A] \\
&\omega_0[x_B] \\
&\omega_0[y_B] \\
&\omega_1[x_A] \rightarrow c_1 \\
&\omega_1[y_A] \rightarrow c_2 \\
&\omega_1[x_B] \rightarrow c_3 \\
&\omega_1[y_B] \rightarrow c_4
\end{align*}
\]
Sites A and B partition into different components after $T_0$ executes. In $H_9$, we allowed queries $T_2$ and $T_4$ to read $x$ and $y$ in both partitions. $H_9$ produces a consistent database state, in the sense that update transactions are 1SR. Moreover, each query reads a consistent database state. Yet there is no serial one-copy history equivalent to $H_9$, so $H_9$ is not 1SR.

Whether to allow queries to read all copies in all components is more of a policy issue than a technical one. For most applications the level of data consistency provided by this technique is probably satisfactory. Although anomalies like $H_9$ are disturbing, the increased data availability to queries will often be the overriding consideration.

Application-dependent Techniques

Site quorums, quorum consensus, and virtual partitions all have the disadvantage of possibly restricting access to copies that are not down. While this restriction is needed to attain one-copy serializability, it does have a cost in lost opportunities. That is, the DBS user is unable to execute certain transactions, and may thereby lose the opportunity to perform certain functions in the real world, resulting in lost revenue, unhappy customers, higher inventory costs, etc. One has to balance these opportunity costs against the value of database integrity. If these costs are high enough, some loss of correctness may be tolerable.

It may therefore be advisable to allow transactions to operate on any available data, even at the risk of non-1SR execution during a communications failure. After the failure is repaired, a special recovery process is invoked to fix database inconsistencies. To make this approach practical for even moderately sized databases, the recovery process needs to be automated.

For example, the recovery process could analyze the logs from sites in different components to find read-write conflicts between transactions in different components. To produce a correct database state, the recovery process could undo and reexecute those transactions as needed. This might entail reexecuting a transaction that committed before the partition (e.g., $T_1$ or $T_2$ in $H_9$), causing it to produce different results than its first execution, a violation of the definition of commitment. Alternatively, the recovery process might only find database inconsistencies by performing validation checks on the data items themselves. Once found, those inconsistencies could be repaired by human intervention, perhaps with the help of a rule-based system.

These approaches may be essential for attaining any appropriately high level of data availability. However, since they allow non-1SR executions to occur, the solutions are necessarily application dependent, and therefore require a highly trained application engineering staff. For users that lack such expertise, general purpose solutions that ensure 1SR executions are usually preferable.
8.9 THE QUORUM CONSENSUS ALGORITHM

In the quorum consensus (QC) algorithm, we assign a non-negative weight to each copy \( x_A \) of \( x \). We then define a read threshold \( RT \) and write threshold \( WT \) for \( x \), such that both \( 2 \cdot WT \) and \( (RT + WT) \) are greater than the total weight of all copies of \( x \). A read (or write) quorum of \( x \) is any set of copies of \( x \) with a weight of at least \( RT \) (or \( WT \)). (Note that the quorums defined here are copy quorums, as opposed to the site quorums of the previous section.) The key observation is that each write quorum of \( x \) has at least one copy in common with every read quorum and every write quorum of \( x \).

In QC, the TM is responsible for translating Reads and Writes on data items into Reads and Writes on copies. A TM translates each \( \text{Write}(x) \) into a set of \( \text{Writes} \) on each copy of some write quorum of \( x \). It translates each \( \text{Read}(x) \) into a set of \( \text{Reads} \) on each copy of some read quorum of \( x \), and it returns to the transaction the most up-to-date copy that it read.

To help the TM figure out which copy is most up-to-date, we tag each copy with a version number, which is initially 0. When the TM processes \( \text{Write}(x) \), it determines the maximum version number of any copy it is about to write, adds one to it, and tags all of the versions that it writes with that version number. Clearly, this requires reading all of the copies in the write quorum before writing any of them. This can be done by having \( \text{Write}(x_A) \) return its version number to the TM. The TM can send the new value and new version number piggybacked on the first round of messages of the atomic commitment protocol.

The version numbers measure how up-to-date each copy is. Each \( \text{Read} \) of a copy returns its version number along with its data value. The TM always selects a copy in the read quorum with the largest version number (there may be more than one such copy but they will all have the same value).

The purpose of quorums is to ensure that Reads and Writes that access the same data item also access at least one copy of that data item in common. This avoids the problems we saw in histories \( H_1, H_2, \) and \( H_3 \). Even if some copies are down and are therefore unavailable to Reads and Writes, as long as there are enough copies around to get a read quorum and write quorum, transactions can still continue to execute. Every pair of conflicting operations will always be synchronized by some scheduler, namely, one that controls access to a copy in the intersection of their quorums.

QC works with any correct concurrency control algorithm. As long as the algorithm produces serializable executions, QC will ensure that the effect is just like an execution on a single copy database. To see why, consider any serial execution equivalent to the actual interleaved execution. If a transaction \( T_i \) reads a copy of data item \( x \), it reads it from the last transaction before it, \( T_j \), that wrote any copy of \( x \). This is because \( T_i \) wrote a write quorum of \( x \), \( T_j \) read a read quorum of \( x \), and every read and write quorum has a nonempty intersection. And since \( T_i \) is the last transaction that wrote \( x \) before \( T_j \) read it, \( T_i \)
placed a bigger version number on all of the copies of $x$ it wrote than the version number written by any transaction that preceded it. This ensures that $T_j$ will read the value written by $T_i$, and not by some earlier transaction.

A nice feature of QC is that recoveries of copies require no special treatment. A copy of $x$ that was down and therefore missed some Writes will not have the largest version number in any write quorum of which it is a member. Thus, after it recovers, it will not be read until it has been written at least once. That is, transactions will automatically ignore its value until it has been brought up-to-date.

Unfortunately, QC has some not so nice features, too. Except in trivial cases, a transaction must access multiple copies of each data item it wants to read. Even if there is a copy of the data item in the DM at the site at which it is executing, the transaction still has to look elsewhere for other copies so it can build a read quorum. In many applications, transactions read more data items than they write. Such applications may not perform well using QC.

One might counter this argument by recommending that each read quorum of $x$ contain only one copy of $x$. But then there can only be one write quorum for $x$, one that contains all copies of $x$. This would lead us to the write-all approach in Section 8.1, which we found was unsatisfactory.

A second problem with QC is that it needs a large number of copies to tolerate a given number of site failures. For example, suppose quorums are all majority sets. Then QC needs three copies to tolerate one failure, five copies to tolerate two failures, and so forth. In particular, two copies are no help at all. With two copies QC can't even tolerate one failure.

A third problem with quorum consensus is that all copies of each data item must be known in advance. A known copy of $x$ can recover, but a new copy of $x$ cannot be created because it could alter the definition of $x$'s quorums. In principle, one can change the weights of the sites (and thereby the definition of quorums) while the DBS is running, but this requires special synchronization (see Exercise 8.20).

We will see other approaches to replicated data concurrency control that circumvent QC's weaknesses. Before moving on, though, let's define the histories that QC produces and prove that they are all 1SR.

*Correctness Proof for Quorum Consensus*

A QC history is an RD history that models an execution of the QC algorithm. Every QC history $H$ has the following properties.

**QC:** If $T_i$ writes $x$, then $H$ contains $w_i[x_{A_1}], \ldots, w_i[x_{A_n}]$ for some write quorum $w_q(x) = \{x_{A_1}, \ldots, x_{A_n}\}$ of $x$.

**QC** is simply the rule for using write quorums.

Let $\text{last}_i(x) = \{T_j\} \text{ for some } x_A \in w_q(x), w_i[x_{A}] \text{ is the last Write on } x_A \text{ that precedes } w_i[x_{A}] \text{ in } H\}$. Notice that $\text{last}_i(x) = \{\}$ if $T_i$ is the first transaction in
$H$ to write $x$. Define $T_i$'s version number for $x$ to be $\text{VN}_i(x) = 1 + \max\{\text{VN}_j(x) \mid T_j \in \text{last}_i(x)\}$, where $\max\{} = 0$. Intuitively, $\text{VN}_i(x)$ is the version number that $T_i$ assigns to the copies of $x$ that it writes.

We have to distinguish between the Reads that a transaction applies to the database, and the real Read that is actually selected from among those Reads, that is, the one with the largest version number. We will use $rr_i[x]$, for “real Read,” to represent the Read that is selected.

$\text{QC}_2$: If $T_j$ reads $x$, then $H$ contains $r_j[x_{A_1}], \ldots, r_j[x_{A_n}]$ for some read quorum $rq(x) = \{x_{A_1}, \ldots, x_{A_n}\}$ of $x$. For each $x_B$ in $rq(x)$, let $\text{VN}(x_B) = \text{VN}_i(x)$ where $w_i[x_B]$ is the last Write on $x_B$ that precedes $r_j[x_B]$ in $H$. Then $H$ contains $rr_j[x_{A_i}]$ for some $x_{A_i}$ in $rq(x)$ where $\text{VN}(x_{A_i}) = \max\{\text{VN}(x_B) \mid x_B \in rq(x)\}$. Moreover, $r_j[x_{A_i}] < rr_j[x_{A_i}]$ for each $x_{A_i}$ in $rq(x)$.

$\text{QC}_3$ says that each transaction that reads $x$ reads a read quorum of $x$ and selects from that read quorum a copy with a maximal version number. Since the TM cannot determine the real Read until it knows the version numbers of all copies in $rq(x)$, all Reads of those copies must precede $rr_j[x_{A_i}]$.

$\text{QC}_4$: Every $r_j[x_{A_i}]$ follows at least one $w_i[x_{A_i}], i \neq j$.

$\text{QC}_5$: $\text{SG}(H)$ is acyclic.

$\text{QC}_6$ requires that each copy be initialized before it can be read. $\text{QC}_7$ says that the schedulers use a correct concurrency control algorithm.

For example, consider a database with data items $x$ and $y$, with copies $x_{A_1}, x_{B_1}, x_{C_1}, y_{D_1}, y_{E_1}$, and $y_{F_1}$. Let the read and write quorums be all majority sets. Consider transactions $\{T_0, T_1, T_2\}$:

$T_0 = w_0[x] \rightarrow c_0 \rightarrow w_0[y]$  

$T_1 = r_1[x] \rightarrow w_1[y] \rightarrow c_1$  

$T_2 = r_2[y] \rightarrow w_2[x] \rightarrow c_2$

A possible QC history over these transactions is

In $H_{10}$, $\text{last}_0(x) = \text{last}_0(y) = \{\}$, hence $\text{VN}_0(x) = \text{VN}_0(y) = 0$; $\text{last}_1(y) = \text{last}_0(x) = \{T_0\}$, hence $\text{VN}_1(y) = \text{VN}_0(x) = 1$. The copies of $x$ that $T_1$ reads were both written by $T_0$, hence have identical VNs, and $T_1$ may use the value of either. The copies of $y$ that $T_1$ reads were written by different transactions, hence have different VNs; $T_2$ uses the value of copy $y_{E_1}$, which has the larger
8.9 THE QUORUM CONSENSUS ALGORITHM

VN, as required. The inclusion of both \( r_i[x_A] \) and \( rr_i[x_A] \) does not signify that \( T_i \) reads \( x_A \) twice; it merely means that \( x_A \) is determined to be the copy to be used by \( T_i \) as the value of \( x \).

We will prove that every QC history \( H \) is 1SR by proving that \( SG(H) \) is an RDSG of \( H \). Since \( SG(H) \) is acyclic by QC\(_s\), \( H \) is 1SR by Theorem 8.4.

**Lemma 8.8:** Let \( H \) be a QC history. \( SG(H) \) induces a write order for \( H \).

*Proof:* Let \( T_i \) and \( T_k \) write \( x \). Since all write quorums of a data item intersect, there exists a copy \( x_A \) that \( T_i \) and \( T_k \) both write. These Writes on \( x_A \) conflict, so \( SG(H) \) must have an edge connecting \( T_i \) and \( T_k \). Thus, \( T_i \rightarrow T_k \) or \( T_k \rightarrow T_i \), so \( SG(H) \) induces a write order. \( \Box \)

**Lemma 8.9:** Let \( H \) be a QC history. \( SG(H) \) induces a read order for \( H \).

*Proof:* Suppose \( T_j \) reads \( x \) from \( T_i \), \( T_k \) writes \( x \) (i.e., \( i \neq k \) and \( j \neq k \)), and \( T_i \rightarrow T_k \). We need to prove \( T_j \rightarrow T_k \). By QC\(_a\), \( T_j \) reads a read quorum of \( x \); by QC\(_s\), \( T_k \) writes a write quorum of \( x \). Since any two such quorums must intersect, there is a copy of \( x \), say \( x_B \), such that \( r_j[x_B] \) and \( w_k[x_B] \) are in \( H \). Since these are conflicting operations, either \( r_j[x_B] < w_k[x_B] \) or \( w_k[x_B] < r_j[x_B] \). In the former case, \( T_j \rightarrow T_k \) is in \( SG(H) \) and we are done. We now prove that the latter case leads to contradiction.

Let \( w_k[x_B] \) be the last Write on \( x_B \) preceding \( r_j[x_B] \) (possibly \( h = k \)). Since \( T_j \) reads \( x \) from \( T_i \), by QC\(_a\) \( VN_h(x) \subseteq VN_i(x) \).

We claim that \( VN_k(x) \subseteq VN_h(x) \). If \( h = k \), \( VN_k(x) = VN_h(x) \). If \( h \neq k \), let \( w_i[x_B] < w_{h_1}[x_B] < \ldots < w_{h_m}[x_B] < w_h[x_B] \) be the sequence of Writes on \( x_B \) between \( w_k[x_B] \) and \( w_h[x_B] \). For each pair of adjacent Writes in the sequence, say \( w_{f}[x_B] < w_{g}[x_B] \), by the definition of VN we have \( VN_f(x) < VN_g(x) \). So in this case \( VN_k(x) < VN_h(x) \). Therefore, \( VN_k(x) \subseteq VN_h(x) \) as claimed. Since \( VN_k(x) \subseteq VN_h(x) \), we also have \( VN_k(x) \subseteq VN_i(x) \).

Since \( T_i \) and \( T_k \) write \( x \), by QC\(_s\) they write some copy \( x_C \) in common. Since \( T_i \rightarrow T_k \) and \( SG(H) \) is acyclic (by QC\(_s\)), it must be that \( w_i[x_C] < w_k[x_C] \). Applying the argument of the previous paragraph, we get \( VN_i(x) < VN_k(x) \). But this contradicts \( VN_k(x) \subseteq VN_i(x) \). \( \Box \)

**Theorem 8.10:** The quorum consensus algorithm is correct.

*Proof:* It is enough to show that any history \( H \) that satisfies QC\(_s\) is 1SR. By Lemmas 8.8 and 8.9, \( SG(H) \) is an RDSG of \( H \). By QC\(_s\), \( SG(H) \) is acyclic. By Theorem 8.4, then, \( H \) is 1SR. \( \Box \)

**The Missing Writes Algorithm**

Quorum consensus pays for its resiliency to communications failures by increasing the cost of Reads and by increasing the required degree of replica-
tion. These costs are high if communications failures are infrequent. It would be preferable if the cost could be reduced during reliable periods of operation, possibly at the expense of higher overhead during unreliable periods.

The missing writes algorithm is one approach that exploits this trade-off. During a reliable period, the DBS processes Read(x) by reading any copy of x and Write(x) by writing all copies of x. When a failure occurs, the DBS resorts to QC. After the failure is repaired, it returns to reading any copy and writing all copies. Thus, it only pays the cost of QC during periods in which there is a site or communications failure.

Each transaction executes in one of two modes: normal mode, in which it reads any copy and writes all copies, or failure mode, in which it uses QC. A transaction must use failure mode if it is “aware of missing writes.” Otherwise, it can use normal mode.

A transaction is aware of missing writes (MWs) if it knows that a copy xA does not contain updates that have been applied to other copies of x. For example, if a transaction sends a Write to xA but receives no acknowledgment, then it becomes aware of MWs. More precisely, transaction T is aware of an MW for copy xA in some execution if either T writes x but is unable to write xA, or some transaction Tj is aware of an MW for xA and there is a path from Tj to Ti in the SG of that execution.

Suppose T is aware of an MW for xA. Then the failure of xA must precede T. But if T reads xA, then T precedes the failure of xA. Since T has an inconsistent view of the failure of xA, it risks producing a non-1SR execution. Consequently, if T ever becomes aware of an MW for a copy it read, it must abort.

If T is aware of an MW for xA, it still can read x. But by the preceding argument, it must read a copy of x that isn't missing any Writes. To ensure this, the DBS uses QC. That is, if a transaction that reads (or writes) x is aware of MWs, then it must read (or write) a read (or write) quorum of x.

If T begins running in normal mode and becomes aware of an MW as it runs, the DBS can abort T, and reexecute it in failure mode. Alternatively, it can try to upgrade to failure mode on the fly. That is, for each x that T read, the DBS accesses a read quorum R and checks that the value that T read is at least as up-to-date as all copies in R. For each x that T wrote, the DBS checks that T wrote a write quorum of x (see Exercise 8.21).

To implement the algorithm, we need a mechanism whereby a transaction becomes aware of MWs. If a transaction Ti times out on an acknowledgment to one of its Writes, then it immediately becomes aware of an MW. But what if Tj is aware of an MW and Tj → Ti? The definition of MW awareness requires that T become aware of the MWs that Tj is aware of. To do this, Tj should attach a list L of the MWs it is aware of to each copy yB that it accesses. It tags L to indicate whether it read or (possibly read and) wrote yB. When Tj accesses yB in a mode that conflicts with L's tag, it becomes aware of those MWs. The
DM should acknowledge $T_i$'s access to $y_B$ by returning a copy of $I$. $T_i$ can now propagate $L$, along with other such lists it received, to all of the copies that it accesses. This way a transaction $T_j$ propagates MWs that it's aware of to all transactions that follow $T_j$ in the SG, as required by the definition of MW awareness.

After recovering from failure, the DBS at site $A$ has two jobs to do. First, it must bring each newly recovered copy $x_A$ up-to-date. This is easy to do with a copier transaction. The copier simply reads a quorum of copies of $x$, and writes into all of those copies the most up-to-date value that it read. Version numbers can be used to determine this value, as before.

Second, after a copy $x_A$ has been brought up-to-date, the DBS should delete $x_A$ from the lists of MWs on all copies, so that eventually transactions that access those copies no longer incur the overhead of QC. This entails sending a message to all sites, invalidating entries for $x_A$ on their lists of MWs. The problem is that while these messages are being processed, $x_A$ may fail again and some Write $w_i[x_A]$ may not be applied. Since this MW occurred after the recovery of $x_A$ that caused MW entries for $x_A$ to be invalidated, this MW should not itself be invalidated. However, if the new MW is added to an MW list before the old $x_A$ entry in that list was removed, it risks being removed too.

To avoid this error, entries in MW lists should contain version numbers. We leave the algorithm for properly interpreting these version numbers as an exercise (see Exercise 8.22).

We can prove the correctness of the MW algorithm using RDSGs, much as we did for QC. As in QC, the SG in MW induces a write order. However, it does not necessarily induce a read order, which makes the proof more complex than for QC.

For example, suppose we have three copies \{y_A, x_A, x_B\}, with weight(y_A) = 1, weight(x_A) = 2, and weight(x_B) = 1. The read and write thresholds are: RT(x) = WT(x) = 2, RT(y) = WT(y) = 1. Consider the following history.

\[
H_{11} = w_0[y_A] \rightarrow w_1[y_A] \rightarrow r_3[y_A] \rightarrow r_2[y_A] \rightarrow c_2 \rightarrow c_1 \rightarrow w_0[x_A] \rightarrow r_3[x_A] \rightarrow r_2[x_A] \rightarrow c_2 \rightarrow c_1 \rightarrow x_B \rightarrow x_B \rightarrow x_B \rightarrow r_3[x_B]
\]

In $H_{11}$, $T_3$ and $T_4$ are not aware of MWs and therefore run in normal mode. $T_2$ is aware of its MW on $x_B$, and so runs in failure mode. Although $x_B$ recovers before $T_3$ begins, $T_3$ is still aware of the MW on $x_B$ and therefore runs in failure mode.

$T_1$ reads $x$ from $T_0$, $T_2$ writes $x$, and $T_0 \rightarrow T_2$ is in SG($H_{11}$). To induce a read order, we need $T_1 \rightarrow T_2$ in RDSG($H_{11}$). But $T_1 \rightarrow T_2$ is not in SG($H_{11}$), so
we need to add it to the SG to make it an RDSG. It turns out that in all such cases where the SG of a history generated by the MW algorithm is missing an edge that is needed to create a read order, we can simply add the necessary edge to the RDSG without creating a cycle. This is the main step in proving every such history has an acyclic RDSG and therefore is 1SR (see Exercise 8.23).

8.10 THE VIRTUAL PARTITION ALGORITHM

As we have seen, the major drawback of the quorum consensus algorithm is that it requires access to multiple, remote copies of $x$ in order to process a $\text{Read}(x)$, even when a copy of $x$ is available at the site where the Read is issued. This defeats one of the motivations for data replication, namely, to make data available "near" the place where it is needed. The missing writes algorithm mitigates this problem by using quorum consensus only when site or communication failures exist. The virtual partition (VP) algorithm, studied in this section, is designed so that a transaction never has to access more than one copy to read a data item. Thus, the closest copy available to a transaction can always be used for reading.

As in quorum consensus, each copy of $x$ has a non-negative weight, and each data item has read and write thresholds ($RT$ and $WT$, respectively) with the properties described at the beginning of Section 8.9. Read and write quorums are defined as before. However, these serve a different purpose in VP than in QC, as we'll see presently.

The basic idea of VP is for each site $A$ to maintain a view, consisting of the sites with which $A$ "believes" it can communicate. We denote this set by $v(A)$. As usual, each transaction $T$ has a home site, $\text{home}(T)$, where it was initiated. The view of a transaction, $v(T)$, is the view of its home site, $v(\text{home}(T))$, at the time $T$ starts. As we will see shortly, if $v(\text{home}(T))$ changes before $T$ commits, then $T$ will be aborted.

A transaction $T$ executes as if the network consisted just of the sites in its view. However, for the DBS to process a $\text{Read}(x)$ (or $\text{Write}(x)$) of $T$, $v(T)$ must contain a read (or write) quorum for $x$. If that is not the case, the DBS must abort $T$. Note that the DBS can determine if it can process $\text{Read}(x)$ or $\text{Write}(x)$ from information available at $\text{home}(T)$. It need not actually try to access a read or write quorum of $x$'s copies.

Within the view in which $T$ executes, the DBS uses the write-all approach. That is, it translates $\text{Write}(x)$ into writes on all copies of $x$ in $v(T)$; it translates $\text{Read}(x)$ into a Read of any copy of $x$ in $v(T)$.

A good illustration of how this idea works is provided if we examine how VP avoids $H_{-}$ — our archetype of a problematic history in the presence of communication failures.
Recall that in this example, the network is partitioned into two components, \{A, B\} and \{C, D\}. Suppose \textit{home}(T_1) = A and \textit{home}(T_2) = C. If each site has a correct view of the sites with which it can communicate, \(v(A) = \{A, B\}\) and \(v(C) = \{C, D\}\). If the DBS allows \(T_1\) to commit in \(H_r\), \(v(A)\) must have a read quorum of \(y\) (and a write quorum of \(x\)), and therefore \(v(C)\) can’t have a write quorum of \(y\) (or a read quorum of \(x\)). Hence the VP algorithm will not allow \(T_1\) to commit.

Views of sites change, as site and communication failures cause the network to be reconfigured. Hence, a transaction \(T\) may attempt to communicate with a site whose view is different from \(T\)’s. Or, \(T\) may attempt but fail to communicate with a site in its view. (A transaction communicates with a site to access a copy stored there or send a \textsc{vote-rec} during its commitment). These situations indicate that \textit{home}(\(T\))’s view is not accurate. \(T\) must abort and \textit{home}(\(T\))’s view must be adjusted, as explained next. Moreover, whenever a site \(A\)’s view changes, all active transactions whose home is \(A\) must abort. They can all then be restarted and try to run within \(A\)’s new view.

Maintaining the views in a consistent manner is not a simple matter, because site and communication failures occur spontaneously. VP surmounts this difficulty by using a special View Update transaction. This is issued by the DBS itself, rather than by users. In spirit it is analogous to the Include and Exclude transactions used by the directory-oriented available copies algorithm to maintain a consistent view among all sites of the copies and directories that are available (cf. Section 8.7).

When a site detects a difference between its present view and the set of sites it can actually communicate with, it initiates a View Update transaction whose purpose is:

1. to adjust the views of all the sites in the new view, and
2. to bring up-to-date all copies of all data items for which there is a read quorum in the new view.

In adjusting the views of the sites in a new view, View Updates must be coordinated to avoid problems like the one exemplified by the following scenario. Consider a network with four sites \(A, B, C, D\) such that at some point \(v(B) = \{B\}\); \(v(D) = \{D\}\), and \(v(A) = v(C) = \{A, C\}\). Later, \(B\) discovers that it can communicate with \(A\) and \(C\) (but not with \(D\)) and, at the same time,
D discovers that it can communicate with A and C (but not with B). Thus B will initiate a View Update to create view \{A, B, C\} and D a similar one to create view \{A, C, D\}. Unless these two activities are properly coordinated, it is possible that A joins the first view and C the second, creating a situation where \(v(A) = v(B) = \{A, B, C\}\) and \(v(C) = v(D) = \{A, C, D\}\). If such inconsistent views were allowed to form, \(H\) would be allowed since the views of A and C (home sites of \(T_1\) and \(T_2\), respectively) would both contain read and write quorums for \(x\) and \(y\).

To avoid such inconsistencies, VP uses a view formation protocol. Associated with each view is a unique view identifier (VID). When a site A wishes to form a new view, say \(v\), it generates a VID, newVID, greater than its present VID. A then initiates a two phase protocol to establish the new view:

1. A sends a JOIN-VIEW message to each site in \(v\) and waits for acknowledgments. This message contains newVID.
2. Receipt of JOIN-VIEW by site B constitutes an invitation for B to join the new view being created by A. B accepts the invitation provided its present VID is less than the newVID contained in JOIN-VIEW; otherwise, it rejects the invitation. Accepting the invitation means sending a positive acknowledgment; rejecting it means sending a negative acknowledgment (or nothing at all).
3. After receiving acknowledgments, A can proceed in one of two ways. It may abort the creation of the new view (either by sending explicit messages to that effect or by not sending any messages at all). In this event, A may attempt to restart the protocol using a greater newVID, hoping it will convince more sites to accept the invitation. Alternatively, A may ignore any rejected invitations and proceed to form a new view consisting only of the set of sites \(v' (v' \subseteq v)\) that accepted the invitation. It does this by sending a VIEW-FORMED message to each site in \(v'\) and attaching the set \(v'\) to that message. Any site that receives that message adopts \(v'\) as its view and newVID (which it had received in the JOIN-VIEW message) as its present VID. It need not acknowledge the receipt of VIEW-FORMED.

Returning to our example, if this protocol is followed, A and C will make a consistent choice: Both will join either the view initiated by B (\(\{A, B, C\}\)) or the view initiated by D (\(\{A, C, D\}\)), depending on which of the two had a greater VID. A third possibility, namely, that neither A nor C joins either new view, would obtain if the present VID of A and C is greater than the VIDs of the views B and D are attempting to create.

To ensure network-wide uniqueness, VIDs are pairs \((c, s)\) where \(s\) is a site identifier and \(c\) is a counter stored at \(s\) and incremented each time \(s\) tries to

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\(^{18}\)Recall that timeout failures can give rise to such anomalous situations (cf. Section 7.2).
create a new view. Thus VIDs created by different sites differ in the second com-
ponent, while VIDs created by the same site differ in the first component. Pairs
are compared in the usual way: $(c, s) < (c', s')$ if $c < c'$, or $c = c'$ and $s < s'$.
When a new view is created, all copies of data items for which there is a
read quorum in the view must be brought up-to-date. This is because, in the
new view, any such copy might be read. The task of updating such copies is
also carried out by the View Update transaction. It is done as follows. When a
site $A$ creates a new view $v$, it reads a read quorum of each data item $x$ (for
which there is a read quorum in $v$). It then writes all copies of $x$ in $v$, using the
most recent value it read. Version numbers can be used to determine the most
recent value, as in QC. This procedure need not be carried out for copies that
$A$ can tell are up-to-date (see Exercise 8.24). The entire process of updating
copies can be combined with the view formation protocol (see Exercise 8.25).
This whole activity comprises what we called before a View Update transac-
tion. It should be emphasized that the activity of a View Update is carried out
like that of an ordinary (distributed) transaction. In particular, Reads and
 Writes issued in the process of bringing copies of the new view up-to-date are
synchronized using the DBS's standard concurrency control mechanism. In the
VP algorithm we described, the set of copies of each data item is fixed. Adding
new copies of a data item $x$ alters the quorum of $x$. Therefore, as in QC, it
requires special synchronization (see Exercise 8.26).

VP guarantees that an execution is 1SR by ensuring that all transactions
see failures and recoveries in the same order. Informally, we can argue this
as follows.

1. Each transaction executes entirely within a single view.
2. For any two transactions $T_i$ and $T_j$, if $T_i \rightarrow T_j$, then $T_i$ executed in a
   view whose VID is less than or equal to $T_j$'s.
3. From (2), there is a serial history $H$ that is equivalent to the one that
   actually occurred in which transactions in the same view are grouped
   into contiguous subsequences, ordered by their VIDs.

Within a VP, all transactions have the same view of which copies are function-
ing and which are down. So if we imagine site failures and recoveries to occur
at the beginning of each segment in $H$, then all transactions have a consistent
view of those failures and recoveries. And this, as we know, implies that the
execution is 1SR (see Exercise 8.27).

**BIBLIOGRAPHIC NOTES**

Since the earliest work on distributed databases, replication has been regarded as an
important feature [Rothnie, Goodman 77] and [Shapiro, Millstein 77b]. Some early
algorithms include primary site [Alsberg, Day 76] [Alsberg et al. 76], majority
consensus [Thomas 79], primary copy [Stonebraker 79b], and a TO-based write-all-
available algorithm in SDD-1 [Bernstein, Shipman, Rothnie 80] and [Rothnie et al. 80].
The concept of one copy serializability was introduced in [Attar, Bernstein, Goodman 84]. The theory of 1SR histories was developed in [Bernstein, Goodman 86a] and [Bernstein, Goodman 86b].

Available copies algorithms are described in [Bernstein, Goodman 84], [Chan, Skeen 86], and [Goodman et al. 83]. Weak consistency for queries is discussed in [Garcia-Molina, Wiederhold 82]. Majority consensus was generalized to quorum consensus in [Gifford 79]. Other majority and quorum based algorithms are presented in [Breitwieser, Leszak 82], [Herlihy 86], and [Sequin, Sargeant, Wines 79]. The missing writes algorithm was introduced in [Eager 81] and [Eager, Sevcik 83]. The virtual partition algorithm was introduced in [El Abbadi, Skeen, Cristian 85] and [El Abbadi, Toueg 86].

[Davidson, Garcia-Molina, Skeen 85] gives a survey of approaches to replication, including some methods not described in this chapter: [Skeen, Wright 84], which describes a method for analyzing transaction conflicts to determine when a partition cannot lead to a non-1SR execution (see Exercise 8.19); [Davidson 84], which shows how to analyze logs to determine which transactions must be undone after a network partition is repaired; and [Blaustein et al. 83] and [Garcia-Molina et al. 83b], which describe methods for recovering an inconsistent database after a partition is repaired by exploiting semantics of transactions.

**EXERCISES**

8.1 The primary copy approach to the distribution of replicated Writes is useful for avoiding deadlocks in 2PL. Explain why it eliminates deadlocks resulting from write-write conflicts. Is it helpful in reducing other types of deadlock, too?

8.2 Suppose we use 2PL with the primary copy and write-all approaches to the distribution of Writes. Assume there are no failures or recoveries. Must updaters set write locks on non-primary copies? How would you change your answer if queries are allowed to read non-primary copies?

8.3 Consider an algorithm for creating new copies that satisfies the following three conditions:
   a. it produces serializable executions (its RD histories have acyclic SGs),
   b. no transaction can read a copy until it has been written into at least once, and
   c. once a transaction \( T_i \) has written into a new copy \( x_A \), all transactions that write into \( x \) and come after \( T_i \) in the SG also write into \( x_A \).
   Prove or disprove that such an algorithm produces only 1SR executions.

8.4 Consider the following algorithm for creating a new copy \( x_A \) of \( x \). The DBS uses primary copy for distributing Writes. The DBS produces serializable executions (its RD histories have acyclic SGs). The scheduler that controls \( x_A \) does not allow \( x_A \) to be read until it has been written into
at least once. Once the primary copy's DBS has sent a Write on $x$ to $x_A$, it will send to $x_A$ all subsequent Writes on $x$ that it receives. Assuming that copies never fail, prove that this algorithm produces 1SR histories.

8.5 Suppose a site $A$ fails for a short period. Most data items at $A$ that are replicated elsewhere were probably not updated by any other site while $A$ was down. If other sites can determine the earliest time $\tau$ that $A$ might have failed, then using $\tau$ they can examine their logs to produce a list of data items that were updated while $A$ was down. These are the only data items at $A$ that must be reinitialized when $A$ recovers. Propose a protocol that sites can use to calculate $\tau$.

8.6 One way to avoid copiers on all data items is for a site $B$ to step in as $A$'s spooler after $A$ fails. $B$ spools to a log the Writes destined for $A$ (which $A$ can't process, since it's down). When $A$ recovers, it uses a Restart procedure based on logging to process the log at $B$ (cf. Section 6.4). What properties must $B$'s spooler and $A$'s Restart procedure satisfy for this approach to work correctly? Given those properties, how should $A$ respond to Reads while it is processing $B$'s log?

8.7 A serial RD history $H$ is called 1-serial if, for all $i, j,$ and $x,$ if $T_i$ reads-$x$-from $T_j$ ($i \neq j$), then $T_i$ is the last transaction preceding $T_j$ that writes into any copy of $x$. Prove that if an RD history is 1-serial, then it is 1SR.

8.8* Let $H$ be an RD history over $T$. Let $G(H)$ be a directed graph whose nodes are in one-to-one correspondence with the transactions in $T$. $G(H)$ contains a reads-from order if, whenever $T_j$ reads-$x$-from $T_i$, 

- a. if $(i \neq j)$, then $T_i \preceq T_j$, and 
- b. if for some copy $x_A$, $w_x[T_i] < r_x[T_A]$, $w_x[T_A] \in H$ and $T_k \preceq T_i$ then $w_x[T_A] < w_x[T_k]$.

$G(H)$ is a weak replicated data serialization graph (weak RDSG) for $H$ if it contains a write order, a reads-from order, and a read order. Prove that $H$ has the same reads-from relationships as a serial 1C history over $T$ if and only if $H$ has an acyclic weak RDSG. Give an example of an RD history that does not have an acyclic RDSG but does have an acyclic weak RDSG.

Since a weak RDSG need not contain $SG(H)$, Theorem 8.3 does not apply. Thus, we are not justified in dropping final writes from a proof that $H$ is equivalent to a serial 1C history. Prove or disprove that $H$ is 1SR iff it has an acyclic weak RDSG.

8.9* In the definition of FRSGs, we assumed that each copy is created once, fails once, and then never recovers. Suppose we drop the assumption that failed copies never recover. To cope with multiple failures and recoveries of a copy, we extend an FRSG to be allowed to include more than one pair of nodes $\{create[A], fail[A]\}$ for each copy $x_A$, where the subscripts on create and fail are used to relate matching create/fail pairs. Redefine the edge set of an FRSG to make use of these multiple create/fail pairs. Prove Theorem 8.5 for this new definition of FRSG.
8.10* Suppose we add the following set of edges to an FRSG:

\[ E_2' = \{ T \rightarrow \text{fail}[x_A] \mid T \text{ writes } x_A \} \]

An augmented FRSG is an FRSG that includes the edges defined by \( E_2' \).

a. Give an example history that has an acyclic FRSG, but has no acyclic augmented FRSG.

b. Suppose a concurrency control algorithm only produces histories that have acyclic augmented FRSGs. Explain intuitively what effect this has on the state of a copy after it recovers from a failure.

c. Does the available copies algorithm of Section 8.6 produce histories that have acyclic augmented FRSGs? If not, modify the algorithm so that it does.

8.11 Give an example of a non-1SR execution that would be allowed by the simple available copies algorithm of Section 8.6, if the missing writes validation and the access validation steps were done at the same time (i.e., the validation protocol sent the AVAILABLE and UNAVAILABLE messages at the same time).

8.12* Give a proof of correctness for the simple available copies algorithm (Section 8.6) based on Theorem 8.4. That is, list conditions satisfied by histories produced by that algorithm, justify why they must be satisfied, and prove that any history that satisfies them must have an acyclic RDSG.

8.13 When all copies of a data item fail, we say the item has suffered a total failure. In available copies algorithms special care must be taken in recovering from a total failure of a data item: The copier transaction that brings copies up-to-date must read the value of the copy that failed last. Describe methods for recovering from such total failures, for available copies algorithms.

8.14 How do the quorum consensus, missing writes, and virtual partition algorithms handle total failures of data items?

8.15 In the directory-oriented available copies algorithm, a transaction can perform access validation by checking that all directories from which it read contain the copies it accessed. Why is this so? That is, why is it not necessary to access the copies themselves? Also, if the scheduler uses Strict 2PL, show that access validation requires merely to check that the directories read by the transaction are still available (their contents need not be considered).

8.16* Formalize the correctness argument given in Section 8.7 to derive a proof that the directory-oriented available copies algorithm produces only 1SR executions.

8.17* Give a correctness proof for the directory-oriented available copies algorithm, based on Theorem 8.4. That is, prove that any history representing an execution produced by that algorithm must have an acyclic RDSG.
8.18 Describe, in some detail, an algorithm that allows the creation and
destruction of directory copies, using a (replicated) master directory.

8.19* Suppose each site is assigned a set of classes and can only execute
transactions that are in its classes. (See Section 4.2 for a description of
classes.) Assume that the readset of each class contains its writerset.

Suppose the network has partitioned. Define a directed graph $G$ each
of whose nodes represents the occurrence of a given class in a given parti-
tion. For each pair of nodes $C_i, C_j$ where readset($C_i) \cap$ writerset($C_j) \neq \{\}$,
$G$ contains the edge $C_i \rightarrow C_j$ and, if $C_i$ and $C_j$ are in the same partition,
$C_j \rightarrow C_i$. A multipartition cycle in $G$ is a cycle that contains two or more
nodes whose sites are in different partitions.

Assume that the partitioned network experiences no site or communi-
cation failures or recoveries. Suppose the DBS in each partition uses the
write-all approach with respect to the copies in its partition. Prove that if
$G$ has no multipartition cycles, and $SG(H)$ is acyclic for RD history $H$,
then $H$ is 1SR.

8.20 Describe a method whereby the weights of copies can be changed
dynamically in the quorum consensus algorithm.

8.21 In the missing writes algorithm a transaction begins by using the write-
all approach. If it should ever become aware of missing writes, it can abort
and rerun using quorum consensus. Alternatively, it can try to switch to
quorum consensus “on the fly.” Describe, in some detail, the latter option.
(Hint: Consider doing some extra work during the atomic commitment
protocol.)

8.22 Describe, in some detail, a technique whereby missing writes to a copy
are “forgotten” when all such Writes have actually been applied to the
copy. Make sure your algorithm can handle correctly the situation where a
copy misses (again) some updates while (old) missing updates to that copy
are being “forgotten.”

8.23* Give a precise proof of correctness for the missing writes algorithm by
giving a list of properties satisfied by histories that represent executions of
that algorithm and then proving that any history that satisfies these condi-
tions must have an acyclic RDSG.

8.24 Consider the formation of a new view in the virtual partition algo-

8.25 Describe, in some detail, a View Update transaction, by integrating
the view formation protocol and the copy update process. Take into
account your answer to the previous exercise to optimize the latter.
Explore the round and message complexity of a View Update transaction.

8.26 Describe a method whereby the weights of copies can be changed
dynamically in the virtual partition algorithm.

8.27* Give a formal proof of correctness for the virtual partition algorithm.